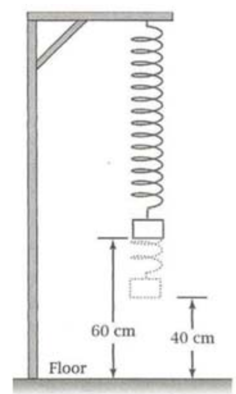
Inverse Trig Word Problems

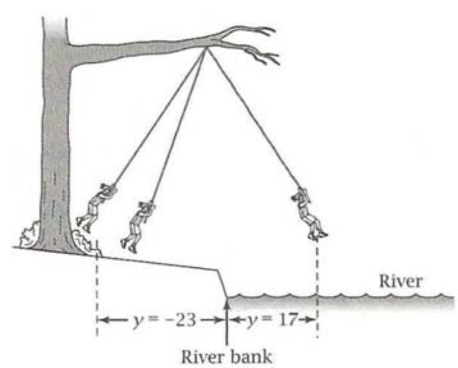
1. Suppose that you are on the beach at Port Aransas, Texas, on August 2. At 2:00 p.m., at high tide, you find that the depth of the water at the end of a pier is 1.5 m. At 7:30p.m., at low tide, the depth of the water is 1.1 m deep. Assume that the depth varies sinusoidally with time.
   a) Find a particular equation for depth as a function of time that has elapsed since 12:00 midnight at the beginning of August 2.
   b) Use your mathematical model to predict the depth of the water at 5:00p.m. on August 3.
   c) At what time does the first low tide occur on August 3?
   d) What is the earliest time on August 3 that the water will be 1.27 m deep?

2. Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time $t = 0$ years. A minimum number of 200 foxes appeared when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t= 5.1$ years.
   a) Sketch the graph of this sinusoid.
   b) Find a particular equation expressing the number of foxes as a function of time.
   c) Predict the fox population when $t = 7, 8, 9,$ and 10 years.
   d) Foxes are declared an endangered species when their population drops below 300. Between what two nonnegative values of t were the foxes first endangered?
   e) Show on your graph in part a that your answer to part d is correct.

3. *Bouncing Spring Problem:* A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. Start a stopwatch. When the stopwatch reads 0.3 sec, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 sec.
   a) Sketch the graph of this sinusoidal function.
   b) Find a particular equation for distance from the floor as a function of time.
   c) What is the distance from the floor when the stopwatch reads 17.2 sec?
   d) What was the distance from the floor when you started the stopwatch?
   e) What is the first positive value of time when the weight is 59 cm above the floor?

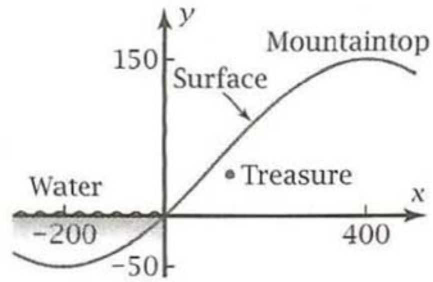


4. Zoey is at summer camp. One day she is swinging on a rope tied to a tree branch, going back and forth alternately over land and water. Nathan starts a stopwatch. When $x = 2$ seconds, Zoey is at one end of her swing, $y = -23$ feet from the river bank. When $x = 5$ seconds, she is then at the other end of her swing, $y = 17$ feet from the river bank. Assume that while she is swinging, y varies sinusoidally with x.
   a) Sketch the graph of y versus x and write the particular equation.
   b) Find y when $x = 13.2$ sec. Was Zoey over land or over water at this time?
   c) Find the first positive time when Zoey was directly over the river bank ($y = 0$).
   d) Zoey lets go of the rope and splashes into the water. What is the value of y for the end of the rope when it comes to rest? What part of the mathematical model tells you this?



5. Suppose you seek a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal vertical cross section. The valley to the left is filled with water to a depth of 50 m, and the top of the mountain is 150 m above the water level. You set up an x-axis at water level and a y-axis 200 m to the right of the deepest part of the water. The top of the mountain is at $x = 400$ m.

   a) Find a particular equation expressing y for points on the *surface* of the mountain as a function of x.
   b) Show algebraically that the sinusoid in part a contains the origin (0, 0).
   c) The treasure is located beneath the surface at the point (130, 40). Which would be a shorter way to dig to the treasure, a horizontal tunnel or a vertical tunnel? Show your work.

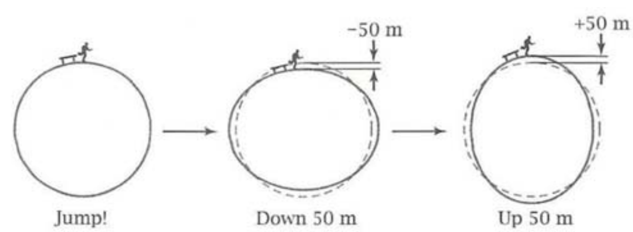


6. For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots," that occur on the surface of the Sun. The number of sunspots in a given year varies periodically, from a minimum of about 10 per year to a maximum of about 110 per year. Between 1750 and 1948 there were exactly 18 complete cycles.

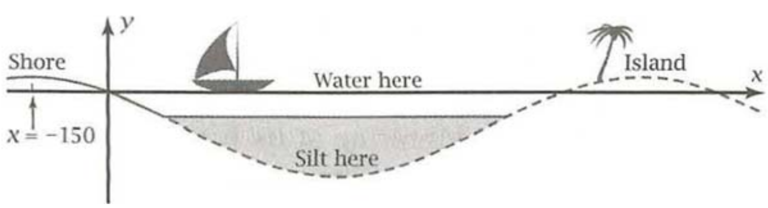
   a) What is the period of a sunspot cycle?
   b) Assume that the number of sunspots per year is a sinusoidal function of time and that a maximum occurred in 1948. Find a particular equation for the number of sunspots per year as a function of the year.
   c) How many sunspots will there be in the year 2020? This year?
   d) What is the first year after 2020 in which there will be about 35 sunspots? What is the first year after 2020 in which there will be a maximum number of sunspots?

7. Suppose that one day all 300+ million people in the United States climb up on tables. At time $t = 0$ they all jump off. The resulting shock wave starts the earth vibrating at its fundamental period of 54 minutes. The surface first moves down from its normal position and then moves up an equal distance above its normal position. Assume that the amplitude is 50m.

   a) Sketch the graph of displacement of the surface from its normal position as a function of time elapsed since the people jumped.
   b) At what time will the surface be its farthest above normal position?
   c) Find a particular equation expressing displacement above normal position as a function of time since the jump.
   d) What is the displacement when $t = 21$?
   e) What are the first three positive times at which the displacement is −37m?



8. Ona Nyland owns an island several hundred feet from the shore of a lake. The figure shows a vertical cross section through the shore, lake, and island. The island was formed millions of years ago by stresses that caused the earth's surface to warp into the sinusoidal pattern shown. The highest point on the shore is at $x = -150$ feet. From measurements on and near the shore (solid part of the graph), topographers find that an equation of the sinusoid is $y = -70 + 100\cos\frac{\pi}{600}(x+150)$ where x and y are in feet. Ona consults you to make predictions about the rest of the graph (dotted).

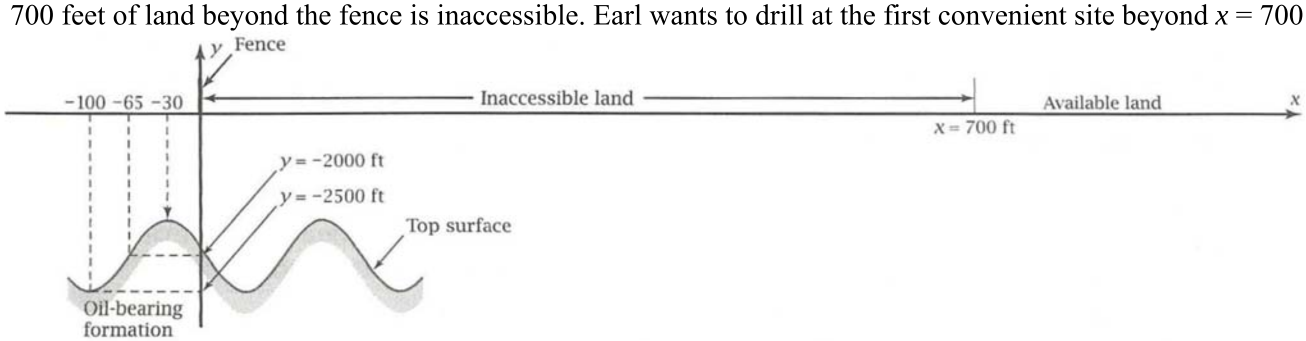


   a) What is the highest the island goes above the water level in the lake? How far from the y-axis is this high point? Show how you got your answers.

b) What is the deepest the sinusoid goes below the water level in the lake? How far from the y-axis is this low point? Show how you got your answers.
c) Over the centuries silt has filled the bottom of the lake so that the water is only 40 feet deep. That is, the silt line is at $y = -40$ feet. Plot the graph. Use a friendly window for x and a window with a suitable range for y. Then find graphically the range of x-values between which Ona would expect to find silt if she goes scuba diving in the lake.
d) If Ona drills an offshore well at $x = 700$ feet, through how much silt would she drill before she reaches the sinusoid? Describe how you got your answer.
e) The sinusoid appears to go through the origin. Does it actually do this, or does it just miss? Justify your answer.
f) Find algebraically the range of x-values between which the island is at or above the water level. How wide is the island, from the water on one side to the water on the other?

9. As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start moving again, the distance between the pebble and the pavement varies sinusoidally with the distance you have gone. The period is the circumference of the tire. Assume that the diameter of the tire is 2 4 in.
a) Sketch the graph of tills sinusoidal function.
b) Find a particular equation for the function. (It is possible to get an equation with *zero* phase displacement.)
c) What is the pebble's distance from the pavement when you have gone 15 in.?
d) What are the first two distances you have gone when the pebble is 11 in. from the pavement?

10. The figure shows a vertical cross section through a piece of land. The y-axis is drawn coming out of the ground at the fence bordering land owned by your boss, Earl Wells. Earl owns the land to the left of the fence and is interested in getting land on the other side to drill a new oil well. Geologists have found an oil-bearing formation below Earl's land that they believe to be sinusoidal in shape. At $x = -100$ feet, the top surface of the formation is at its deepest, $y = -2500$ feet. A quarter-cycle closer to the fence, at $x = -65$ feet, the top surface is only 2000 feet deep. The first 700 feet of land beyond the fence is inaccessible. Earl wants to drill at the first convenient site beyond $x = 700$ feet.



a. Find a particular equation for y as a function of x.
b. Plot the graph on your grapher. Use a friendly window for x of about $[-100, 900]$. Describe how the graph confirms that your equation is correct.
c. Find graphically the first interval of x-values in the available land for which the top surface of the formation is no more than 1600 feet deep.
d. Find algebraically the values of x at the ends of the interval in part c. Show your work.
e. Suppose that the original measurements had been slightly inaccurate and that the value of y shown at −65 feet was at $x = -64$ instead. Would this fact make much difference in the answer to part c? Use the most time-efficient method to reach your answer. Explain what you did.

11. The hum you hear on some radios when they are not tuned to a station is a sound wave of 60 cycles per second.
a) Is the 60 cycles per second the period, or is it the frequency? If it is the period, find the frequency. If it is the frequency, find the period.
b) The *wavelength* of a sound wave is defined as the distance the wave travels in a time equal to one period. If sound travels at 1100 ft./sec, find the wavelength of the 50-cycle-per-second hum.
c) The lowest musical note the human ear can hear is about 16 cycles per second. In order to play such a note, the pipe on an organ must be exactly half as long as the wavelength. What length of organ pipe would be needed to generate a 16-cycle-per-second note?

Answers

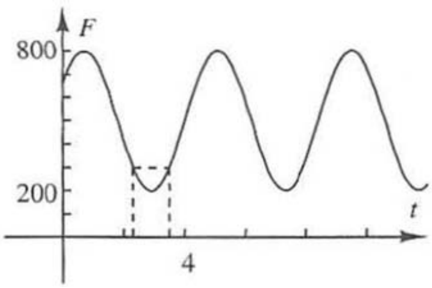
1. a. $d = 1.3 + 0.2\cos\frac{2\pi}{11}(t - 14)$

b. d(41) = 1.108 m

c. 19.5 + 11 = 30.5 = 6:30 a.m. on August 3.

d. 28.014 = 4:00:49 a.m.

2. a.

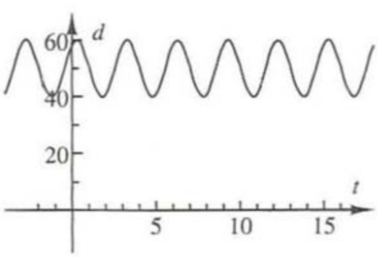


b. $F = 500 - 300\cos\frac{5\pi}{11}(t - 2.9)$

c. F(7) = 227 foxes
F(8) = 338 foxes
F(9) = 727 foxes
F(10) = 727 foxes

d. 2.3 yr < t < 3.5 yr

3. a.



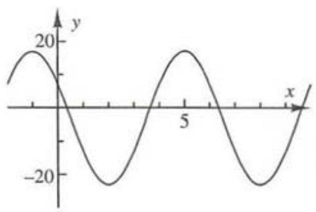
b. $d = 50 + 10\cos\frac{2\pi}{3}(t - 0.3)$

c. 43.309 cm

d. 58.090 cm

e. 0.085 sec

4. a.



$y = -3 + 20\cos\frac{\pi}{3}(x + 1)$

b. 16.383 ft.; Zoey was over land.

c. 0.356 sec

d. y = −3, the sinusoidal axis

5. a. $y = 50 + 100\cos\frac{\pi}{600}(x - 400)$

b. 0 m

c. Vertical tunnel: 25.643 m
Horizontal tunnel: 49.131 m
The vertical tunnel is shorter.

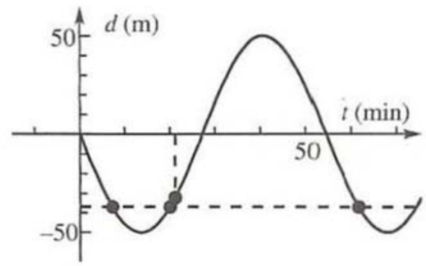
6. a. 11 years

b. $S = 60 + 50\cos\frac{2\pi}{11}(t - 1948)$

c. 12 sunspots

d. 2021.333; maximum in 2025.

7. a.



b. t = $\frac{3}{4}$ period = 40.5 min

c. $d = -50\sin\frac{\pi}{27}t$

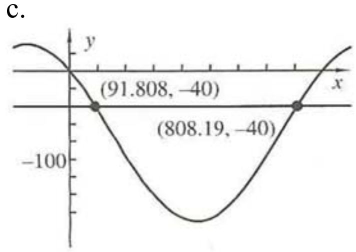
d. −32.1 m

e. 7.160 min, 19.840 min, 61.160 min

8. a. y_{max} = 30 ft. at x = 1050 ft.

b. y_{min} = −170 ft. at 450 ft.

c.



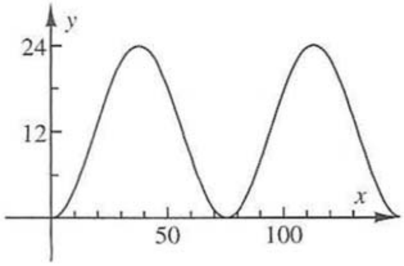
So silt is where 91.808 ft. ≤ x ≤ 808.192 ft.,

d. 55.882 ft.

e. 0.711 ft.; curve just misses the origin.

f. 898.090 ft. ≤ x ≤ 1201 .910 ft., 303.820 ft. wide

9. a.



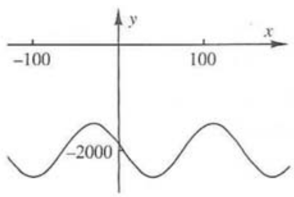
b. $y = 12 - 12\cos\frac{1}{12}x$

c. 8.216 in.

d. 17.848 in. and 57.550 in.

10. a. $y = -2000 + 500\cos\frac{\pi}{70}(x+30)$

b.



c. 795.662 ft.

d. 795.662 ft. $\le$ x $\le$ 824.338 ft.

e. The new equation is

$y = 2000 + 500\cos\frac{\pi}{72}(x+28)$ so now 700 ft. $\le x$ $\le$ 706.748 ft. This is a very small region to drill.

11. a. Frequency = 60 cycles/sec; period = $\frac{1}{60}$ sec

b. Wavelength = $\frac{1100}{60}$ = 18.333 ft. = 220 in.

c. 34.375 ft. = 34 ft. 4.5 in.