Trig Proofs – Sum/Difference

Assume that angle A and B are in standard position, $\sin A = \frac{1}{2}$, $\cos A > 0$, $\tan B = \frac{3}{4}$, and $\sin B < 0$. Draw A and B and then find the following:

1. $\cos(A - B)$
2. $\sin(A - B)$
3. $\sin(A + B)$
4. $\cos(A + B)$
5. $\tan(A - B)$
6. $\tan(A + B)$

Find the exact value of the following using clever choices of your special angles on the unit circle:

7. $\cos 15^\circ$
8. $\sin 15^\circ$
9. $\tan 15^\circ$
10. $\cot 15^\circ$
11. $\sec 15^\circ$
12. $\csc 15^\circ$
13. $\cos 165^\circ$
14. $\sin 165^\circ$
15. $\tan 165^\circ$
16. $\cot 165^\circ$
17. $\sec 165^\circ$
18. $\csc 165^\circ$

Prove the following identities:

19. $\sin(x + 60^\circ) - \cos(x + 30^\circ) = \sin x$
20. $\sin(x + 30^\circ) + \cos(x + 60^\circ) = \cos x$
21. $\tan\left(x + \frac{\pi}{4}\right) + 1 = \sqrt{2}\cos x \sec\left(x + \frac{\pi}{4}\right)$
22. $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \cos x + \sin x$
23. $(\cos A \cos B - \sin A \sin B)^2 + (\sin A \cos B + \cos A \sin B)^2 = 1$
24. $\sin\frac{3x}{7}\cos\frac{4x}{7} + \cos\frac{3x}{7}\sin\frac{4x}{7} = \sin x$

Answers:

1. $\frac{-4\sqrt{3}-3}{10}$
2. $\frac{-4+3\sqrt{3}}{10}$
3. $\frac{-4-3\sqrt{3}}{10}$
4. $\frac{-4\sqrt{3}+3}{10}$
5. $\frac{25\sqrt{3}-48}{39}$
6. $\frac{25\sqrt{3}+48}{39}$
7. $\frac{\sqrt{6}+\sqrt{2}}{4}$
8. $\frac{\sqrt{6}-\sqrt{2}}{4}$
9. $2-\sqrt{3}$
10. $2+\sqrt{3}$
11. $\sqrt{6}-\sqrt{2}$
12. $\sqrt{6}+\sqrt{2}$
13. $\frac{-\sqrt{2}-\sqrt{6}}{4}$
14. $\frac{\sqrt{6}-\sqrt{2}}{4}$
15. $\sqrt{3}-2$
16. $-\sqrt{3}-2$
17. $\sqrt{2}-\sqrt{6}$
18. $\sqrt{6}+\sqrt{2}$