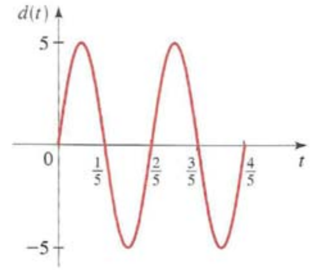
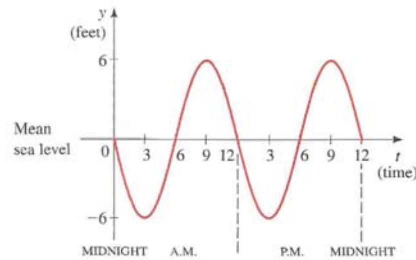
Modeling Trig Functions

1. A mass attached to a spring is moving up and down in simple harmonic motion. The graph gives its displacement $d(t)$ from equilibrium at time t. Determine the equation.

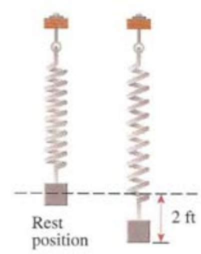


2. The graph shows the variation of the water level relative to mean sea level in Commencement Bay at Tacoma, Washington, for a particular 24-hour period. Assuming that this variation is modeled by simple harmonic motion, find an equation that describes the variation in water level as a function of the number of hours after midnight.



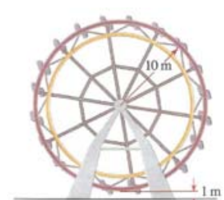
3. The Bay of Fundy in Nova Scotia has the highest tides in the world. In one 12-hour period the water starts at mean sea level, rises to 21 ft. above, drops to 21 ft. below, then returns to mean sea level. Assuming that the motion of the tides is simple harmonic, find an equation that describes the height of the tide in the Bay of Fundy above mean sea level. Determine the equation that shows the level of the tides over a 12-hour period.

4. A mass suspended from a spring is pulled down a distance of 2 ft. from its rest position, as shown in the figure. The mass is released at time $t = 0$ and allowed to oscillate. If the mass returns to this position after 1 s, find an equation that describes its motion.



5. A mass is suspended on a spring. The spring is compressed so that the mass is located 5 cm above its rest position. The mass is released at time $t = 0$ and allowed to oscillate. It is observed that the mass reaches its lowest point 1 s after it is released. Find an equation that describes the motion of the mass.

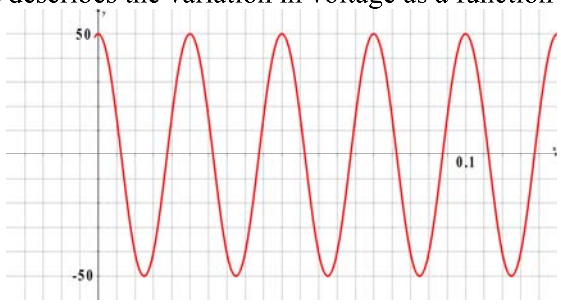
6. A Ferris wheel has a radius of 10 m, and the bottom of the wheel passes 1 m above the ground. If the Ferris wheel makes one complete revolution every 20 s, find an equation that gives the height above the ground of a person on the Ferris wheel as a function of time.



7. The variable star Zeta Gemini has a period of 10 days. The average brightness of the star is 3.8 magnitudes. and the maximum variation from the average is 0.2 magnitude. Assuming that the variation in brightness is simple harmonic, find an equation that gives the brightness of the star as a function of time.

8. Astronomers believe that the radius of a variable star increases and decreases with the brightness of the star. The variable star Delta Cephei has an average radius of 20 million miles and changes by a maximum of 1.5 million miles from this average during a single pulsation. Find an equation that describes the radius of this star as a function of time.

9. The graph shows an oscilloscope reading of the variation in voltage of an AC current produced by a simple generator. Find a formula that describes the variation in voltage as a function of time.



10. The height of the tide in a small beach town is measured along a seawall. Water levels oscillate between 7 feet at low tide and 15 feet at high tide. On a particular day, low tide occurred at 6 AM and high tide occurred at noon. Approximately every 12 hours, the cycle repeats. Find an equation to model the water levels.

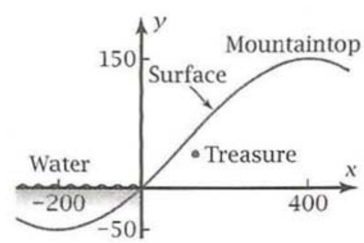
11. Outside temperatures over the course of a day can be modeled as a sinusoidal function. Suppose the high temperature of 105°F occurs at 5 PM and the average temperature for the day is 85°F. Find an equation to model the temperatures.

12. A Ferris wheel is 20 meters in diameter and boarded from a platform that is 2 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 6 minutes. Find an equation to model the height of the person on the Ferris wheel.

13. The sea ice area around the North Pole fluctuates between about 6 million square kilometers on September 1 to 14 million square kilometers on March 1. Assuming a sinusoidal fluctuation, determine a sinusoidal function that models the area using time in months.

14. Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time $t = 0$ years. A minimum number of 200 foxes appeared when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t= 5.1$ years. Find a particular equation expressing the number of foxes as a function of time.

15. Suppose you seek a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal vertical cross section. The valley to the left is filled with water to a depth of 50 m, and the top of the mountain is 150 m above the water level. You set up an *t*-axis at water level and a *y*-axis 200 m to the right of the deepest part of the water. The top of the mountain is at $t = 400$ m. Find a particular equation expressing *y* for points on the *surface* of the mountain as a function of *t*.



Answer Bank:

$y = 12 - 10\cos\frac{\pi}{3}(t)$

$y = 5\cos(\pi t)$

$y = 5\sin(5\pi t)$

$y = 11 + 4\cos\frac{\pi}{6}(t)$

$y = -6\sin\frac{\pi}{6}(t)$

$y = 10 + 4\cos\frac{\pi}{6}(t-3)$

$y = 50 + 100\,cos\frac{\pi}{600}(t-400)$

$y = 21\sin\frac{\pi}{6}(t)$

$y = -2\cos(2\pi t)$

$y = 20 + 1.5\sin 2\pi(t)$

$y = 6 - 5\cos\frac{\pi}{10}(t)$

$y = 3.8 + 0.2\sin\frac{\pi}{5}(t)$

$y = 50\cos 80\pi(t)$

$y = 85 + 20\cos\frac{\pi}{12}(t-5)$

$y = 500 - 300\,cos\frac{5\pi}{11}(t-2.9)$

$y = 11 - 10\sin\frac{\pi}{10}(t)$