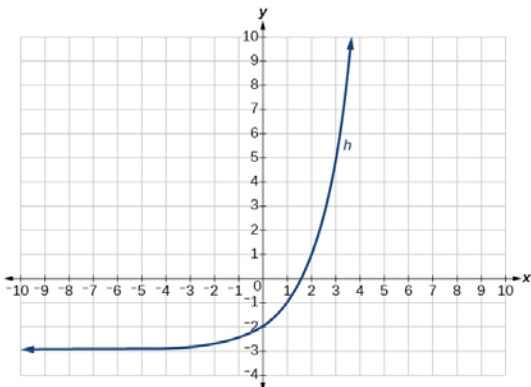


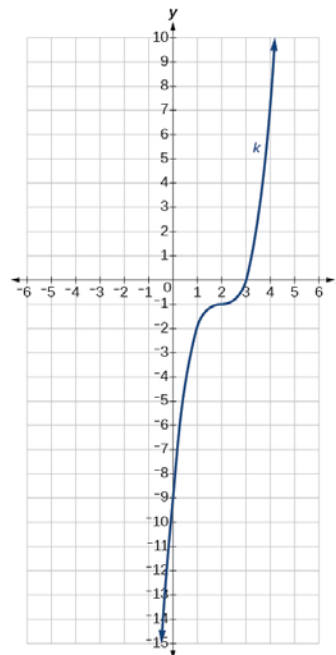
## 1.5 Transformation of Functions

1. A horizontal shift results when a constant is added to or subtracted from the input. A vertical shift results when a constant is added to or subtracted from the output.
3. A horizontal compression results when a constant greater than 1 multiplies the input. A vertical compression results when a constant between 0 and 1 multiplies the output.
5. For a function  $f$ , substitute  $(-x)$  for  $(x)$  in  $f(x)$  and simplify. If the resulting function is the same as the original function,  $f(-x) = f(x)$ , then the function is even. If the resulting function is the opposite of the original function,  $f(-x) = -f(x)$ , then the original function is odd. If the function is not the same or the opposite, then the function is neither odd nor even.
7. A shift down 3 units and to the right 1 unit will result in  $g(x) = f(x-1) - 3$ , which gives us  $g(x) = |x-1| - 3$ .
9. A shift up 2 units and to the left 4 units will result in  $g(x) = f(x+4) + 2$ , which gives us  $g(x) = \frac{1}{(x+4)^2} + 2$ .
11. The graph of  $f(x+43)$  is a horizontal shift to the left 43 units of the graph of  $f$ .
13. The graph of  $f(x-4)$  is a horizontal shift to the right 4 units of the graph of  $f$ .
15. The graph of  $f(x)+8$  is a vertical shift up 8 units of the graph of  $f$ .
17. The graph of  $f(x)-7$  is a vertical shift down 7 units of the graph of  $f$ .
19. The graph of  $f(x+4)-1$  is a horizontal shift to the left 4 units and a vertical shift down 1 unit of the graph of  $f$ .
21. This is a parabola with its vertex shifted from  $(0, 0)$  to  $(-3, -2)$ , so the graph is decreasing on  $(-\infty, -3)$  and increasing on  $(-3, \infty)$ .
23. This is a square root function reflected across the  $x$ -axis, stretched by a factor of 3 and shifted down 1 unit, so it is decreasing on  $(0, \infty)$ .

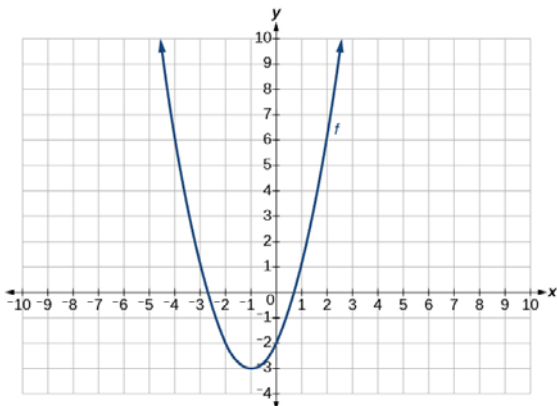
25.



29.



27.



31. The output values for  $g(x)$  have been shifted 1 unit to the right in relation to the input values, so  $g(x) = f(x-1)$ . The output values for  $h(x)$  have all been increased by 1, so  $h(x) = f(x) + 1$ .

33. This is a transformation of the absolute value function 2 units down and 3 units to the right, so  $f(x) = |x - 3| - 2$ .

35. This is a transformation of the square root function 1 unit down and 3 units to the left, so  $f(x) = \sqrt{x + 3} - 1$ .

37. This is a transformation of the parabola function to the right 2 units, so  $f(x) = (x - 2)^2$ .

39. This is a transformation of the absolute value function 2 units down and 3 units to the left, so  $f(x) = |x + 3| - 2$ .

41. This is a reflection of the square root function across the  $x$ -axis, so  $f(x) = -\sqrt{x}$ .

43. This is a transformation of the parabola function. It is reflected across the  $x$ -axis, shifted up 2 units and to the left 1 unit, so  $f(x) = -(x + 1)^2 + 2$ .

45. This is a transformation of the square root function. It is reflected across the  $y$ -axis and shifted up 1 unit, so  $f(x) = \sqrt{-x} + 1$ .

47. Evaluating  $f(-x) = 3(-x)^4 = 3x^4 = f(x)$ , so this function is even.

49. Evaluating  $g(-x) = \frac{1}{-x} + 3(-x) = -\frac{1}{x} - 3x = -\left(\frac{1}{x} + 3\right) = -g(x)$ , so this function is odd.

51. Evaluating  $g(-x) = 2(-x)^4 = 2x^4 = g(x)$ , so this function is even.

53. The graph of  $g$  is a vertical reflection (across the  $x$ -axis) of the graph of  $f$ .

55. The graph of  $g$  is a vertical stretch by a factor of 4 of the graph of  $f$ .

57. The graph of  $g$  is a horizontal compression by a factor of  $\frac{1}{5}$  of the graph of  $f$ .

59. The graph of  $g$  is a horizontal stretch by a factor of 3 of the graph of  $f$ .

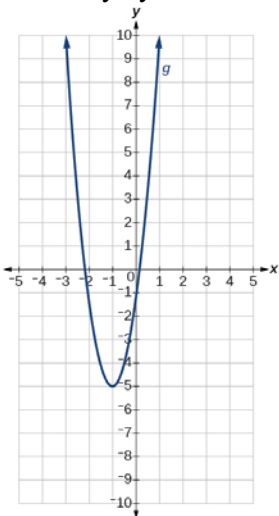
61. The graph of  $g$  is a horizontal reflection across the  $y$ -axis and a vertical stretch by a factor of 3 of the graph of  $f$ .

63. This will result in  $g(x) = f(-4)$ , so  $g(x) = |-4x|$ .

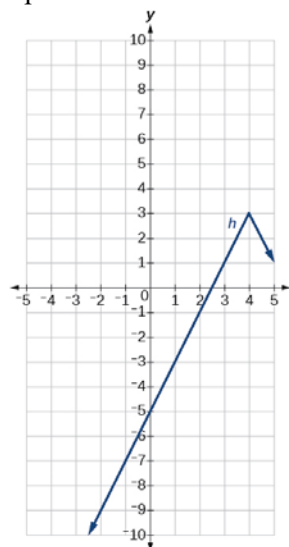
65. This will result in  $g(x) = f(3(x + 2)) - 3$ , so  $g(x) = \frac{1}{3(x + 2)^2} - 3$ .

67. This will result in  $g(x) = \frac{1}{2}f(x - 5) + 1$ , so  $g(x) = \frac{1}{2}(x - 5)^2 + 1$ .

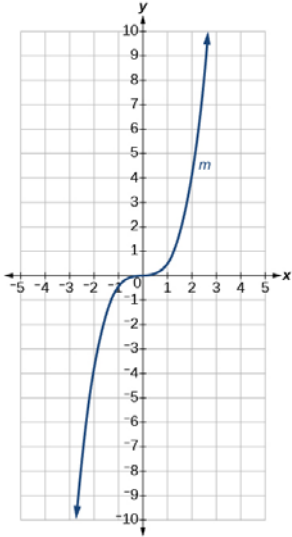
69. This is a parabola shifted to the left 1 unit, stretched vertically by a factor of 4, and shifted down 5 units.



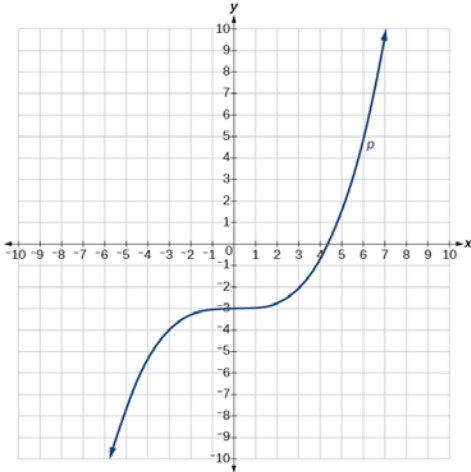
71. This is an absolute value function stretched vertically by a factor of 2, shifted 4 units to the right, reflected across the horizontal axis, and then shifted 3 units up.



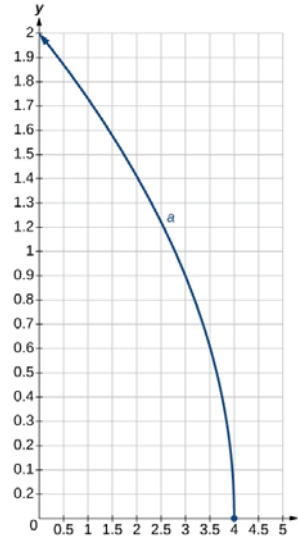
73. This is a cubic function compressed vertically by a factor of  $\frac{1}{2}$ .



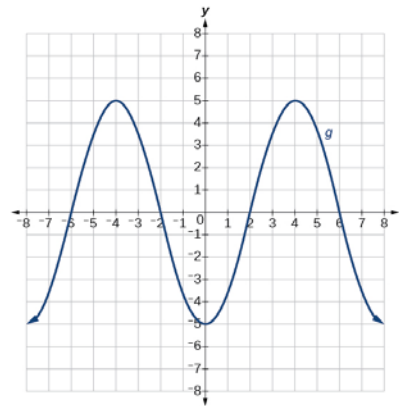
75. The graph of the function is stretched horizontally by a factor of 3 and then shifted downward by 3 units.



77. The graph of  $f(x) = \sqrt{x}$  is shifted 4 units to the right and then reflected across the y-axis.



79. The graph of  $g(x)$  will be the same as the graph of  $f(x)$ , but reflected across the x-axis.



81. The graph of  $g(x)$  will be the same as the graph of  $f(x)$ , but shifted 2 units to the right.

