1.3 Rates of Change and Behavior Graphs

1. Yes, the average rate of change of all linear functions is constant.
3. The absolute maximum and minimum relate to the entire graph, whereas the local extrema relate only to a specific region in an open interval.
5. $4(b+1); b \neq 1$
7. $3; h \neq 0$
9. $4x + 2h; h \neq 0$
11. $\dfrac{-1}{13(13+h)}; h \neq 0$
13. $3h^2 + 9h + 9; h \neq 0$
15. $4x + 2h - 3; h \neq 0$
17. We have two points: $(2,\ 3)$ and $(5,\ 7)$. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-3}{5-2} = \dfrac{4}{3}$
19. The function is increasing on $\left(-\infty, -2.5\right) \cup \left(1, \infty\right)$ and decreasing on $(-2.5,\ 1)$.
21. The function is increasing on $\left(-\infty, 1\right) \cup \left(3, 4\right)$ and decreasing on $\left(1, 3\right) \cup \left(4, \infty\right)$.
23. The function appears to have a local maximum: $(-3,\ 60)$ and a local minimum: $(3, -60)$.
25. The function appears to have an absolute maximum of 150 at $x = 7$ and an absolute minimum of –220 at $x = -7.5$. The function appears to have an absolute maximum at approximately $(7,\ 150)$ and an absolute minimum at approximately $(-7.5,\ -220)$.
27. a. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{77 - 83}{2004 - 2002} = -\dfrac{6}{2} = -3$. So the population has decreased by an average of 3000 per year.

b. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{78 - 83}{2006 - 2002} = \dfrac{-5}{4} = -1.25$. So the population has decreased by an average of 1250 per year.

29. $\dfrac{h(4) - h(-2)}{4 - (-2)} = -4$
31. $\dfrac{g(3) - g(-3)}{3 - (-3)} = 27$
33. $\dfrac{p(1) - p(-3)}{1 - (-3)} = -\dfrac{1}{6}$
35. There is a local minimum at $(3, -22)$. The function is decreasing on $(-\infty,\ 3)$ and increasing on $(3,\ \infty)$.
37. There is a local minimum at $(-2, -2)$. The function is decreasing on $(-3, -2)$ and is increasing on $(-2,\ \infty)$.
39. There is a local maximum at $(-0.5,\ 6)$ and local minima at $(-3.25, -47)$ and $(2.1, -32)$. The function is decreasing on $(-\infty, -3.25)$ and $(-0.5,\ 2.1)$ and increasing on $(-3.25,\ -0.5)$ and $(2.1,\ \infty)$.
41. A. From the graph we can see that the point is a relative (local) maximum of the function.
43. $\dfrac{f(b) - f(2)}{b - 2} = -\dfrac{1}{10} \Rightarrow b = 5$
45. We have two points: $(0,\ 0)$ and $(4,\ 10.7)$. $\dfrac{y_2 - y_1}{x_2 - x_2} = \dfrac{10.7 - 0}{4 - 0} \approx 2.7$. The gas was flowing at approximately 2.7 gallons per minute.
47. We have two points: $(5,\ 12)$ and $(15,\ 6)$. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 12}{15 - 5} = -0.6$. The radioactive substance is decaying at approximately 0.6 milligrams per day.