Pre-Calculus 1st Semester Exam Review

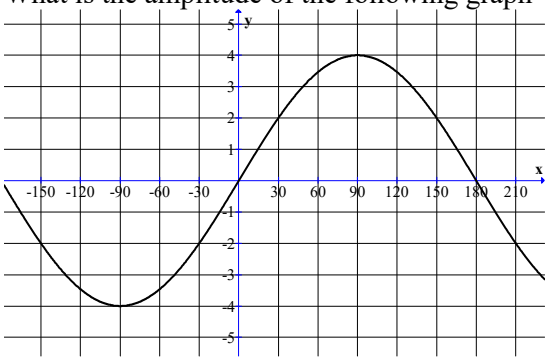
1. Identify the value of $\sin^{-1}(.5)$ (in degrees)
2. $\theta = 325°$, find θ_{ref}
3. $\theta = -185°$, find θ_{ref}
4. $\theta = -142°$, $\theta_{ref} = ?$
5. Convert 710° into radians
6. Convert 135° into radians
7. Convert $\frac{-3\pi}{8}$ into degrees
8. Convert $\frac{2\pi}{9}$ into degrees
9. $\sin\theta = -\frac{2}{3}$ and θ terminates in Quadrant III, $\tan\theta = ?$
10. $\sec\theta = \frac{5}{2}$ and θ terminates in Quadrant IV, find $\csc\theta$
11. $\cos\theta = \frac{1}{3}$ and θ terminates in Quadrant I, find $\cot\theta$
12. If the terminal side of θ passes through (4, – 3), find $\tan\theta$.
13. If $\tan\theta = \frac{5}{7}$ in Quadrant III, find $\csc\theta$
14. Which functions are positive in Quadrant I?
15. Which trigonometric functions are positive in Quadrant II?
16. Which trigonometric functions are positive in Quadrant IV?
17. $y = \csc x$ is positive in Quadrants?

Find:

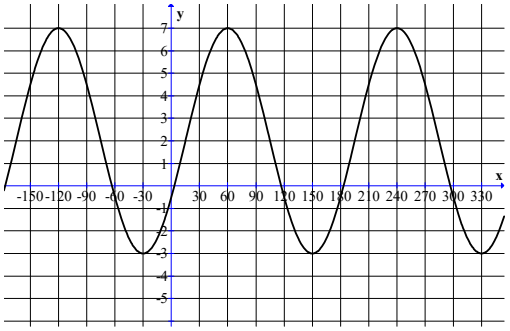
18. $\tan\frac{\pi}{3}$
19. $\sin\left(\frac{5\pi}{6}\right)$
20. $\cos\left(\frac{2\pi}{3}\right)$
21. $\csc\frac{7\pi}{6}$
22. $\sec\frac{3\pi}{4}$
23. $\cos\frac{-\pi}{6}$
24. $\sec\frac{-5\pi}{6}$
25. $\cos\left(-\frac{\pi}{3}\right)$
26. $\cos^2\left(-\frac{\pi}{3}\right) + \sin^2\left(-\frac{\pi}{3}\right)$
27. $4\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{3\pi}{4}\right)$
28. $\cos^2\left(-\frac{\pi}{3}\right) - \sin^2\left(\frac{\pi}{6}\right)$
29. $\sec^2\left(\frac{\pi}{3}\right) - \csc^2\left(\frac{\pi}{4}\right)$
30. Graph $y = 2 + 3\cos\frac{1}{2}(\theta - 20^\circ)$
31. Graph $y = 1 + \tan\theta$
32. Graph $y = -1 + 2\cos 3(\theta - 50^\circ)$

No Calculator for Problems #1 – 71

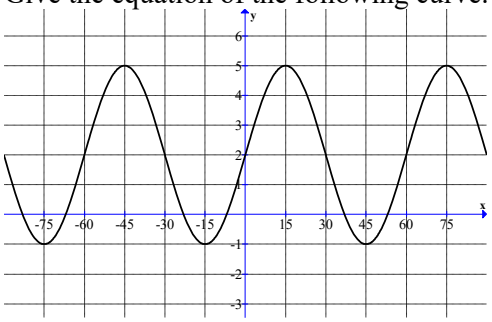
33. Graph $y = 2 + 3\sin 2(\theta - 30^\circ)$
34. What is the amplitude of the following graph



35. What is the horizontal shift if the graph is a sine curve? A cosine curve?



36. Give the equation of the following curve.



37. Identify the period of $y = -2 + 3\cos\frac{1}{4}(\theta - 20^\circ)$
38. What transformation were performed on $y = -2 + 3\cos\left(3x + \frac{\pi}{2}\right)$
39. Identify the equation of a sinusoid whose period is 180°, whose vertical shift is 3, whose amplitude is 6, and whose horizontal shift for *sin* is 100°.
40. Identify the equation of a sinusoid whose period is 120°, whose vertical shift is 2, whose amplitude is 1, and whose horizontal shift for *sin* is 55°.

41. Identify the range of $y = 2 + 3\cos\frac{1}{2}x$
42. Identify the period of $y = 3\sec 5\left(\theta - 30^\circ\right)$
43. Identify the vertical shift of $y = 2 + 3\cos 4x$
44. Identify the horizontal shift of $y = 9 + 7\sin\frac{1}{4}\left(\theta - 100^\circ\right)$
45. Simplify $\frac{\sin^2 x + \cos^2 x}{\tan x}$
46. Show that $\csc x - \sin x = \cot x \cos x$
47. Prove $\frac{\csc^2 x - 1}{\cos x} = \cot x \csc x$
48. Simplify $\cos x + \sin x \tan x$
49. Simplify $\left(1 - \cos x\right)\left(\csc x + \cot x\right)$
50. Simplify $\frac{\cos x \cot x + \sin x}{2\cos x}$
51. Simplify $\cos x \sin x \tan x$
52. Identify the solution set over the given domain: $2\cos^2 x - 3\cos x + 1 = 0$, $[-2\pi, 2\pi]$
53. Identify the solution set over the given domain: $2\csc x - 4 = 0$ for $[0, 2\pi]$
54. Identify the solution set over the given domain: $4\cos^2 x - 1 = 0$ for $[0, 360^\circ]$
55. Identify the solution set over the given domain: $2\sin^2 x + \sin x - 1 = 0$ for $[-\pi, \pi]$

Find:

56. $\arctan\sqrt{3}$
57. $\arcsin\left(\frac{-\sqrt{3}}{2}\right)$
58. $\arccos\left(\frac{-1}{2}\right)$
59. $\csc^{-1}\left(\csc\frac{7\pi}{6}\right)$
60. $\sin\left(\arccos\left(-\frac{8}{17}\right)\right)$
61. $\arccos\frac{\sqrt{3}}{2}$
62. $\operatorname{arcsec}(-2)$
63. $\arcsin\left(\cos\left(-\frac{\pi}{3}\right)\right)$

Find Domain and Range for the following

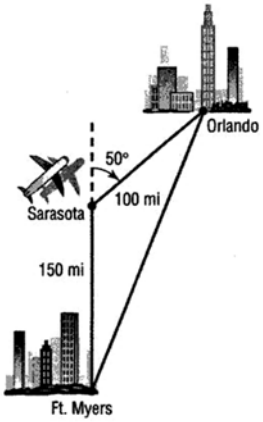
64. $f(x) = \frac{\pi}{2} + 3\sin^{-1} 2(x+1)$
65. $f(x) = \pi + 4\arcsin 3(x-2)$
66. $f(x) = 2 + \frac{3}{\pi}\cos^{-1}\frac{1}{3}(x+2)$
67. $f(x) = 3 + \frac{2}{\pi}\arccos\frac{1}{2}(x-5)$
68. $f(x) = \frac{2\pi}{3} + 2\arctan 10(x-1)$
69. $\sin 75^\circ =$
70. $\sec 195^\circ =$
71. Given with $\sin A = \frac{2}{3}$ with $\frac{\pi}{2} < A < \pi$ and $\sin B = -\frac{1}{2}$ with $\pi < B < \frac{3\pi}{2}$, use identities to find exact values for
    a. $\cos 2A$
    b. $\tan 2B$
    c. $\sin(A+B)$
    d. $\cos(A-2B)$

Calculator is allowed

72. A ship is observed from the top of a lighthouse at an angle of depression of 26°. If the lighthouse is 140 m high, how far is the ship from the base of the lighthouse?
73. An observer at the top of the lighthouse 80 ft. above the surface of the water measures an angle of depression of 0° 42' to a distant ship. How many miles is the ship from the base of the lighthouse?
74. Given ΔABC with $\angle A = 25^\circ$, $b = 10$, and $c = 12$. Find a.
75. Given ΔABC with $\angle A = 33^\circ$, $b = 7$, and $c = 15$. Find area.
76. Given ΔABC with $\angle A = 40^\circ$, $b = 8$, and $c = 10$. Find perimeter.
77. Given ΔABC with $\angle A = 35^\circ$, $\angle B = 70^\circ$, $a = 7$. Find b.
78. Given ΔABC with $a = 10$, $b = 15$, and $c = 20$. Find $\angle B$.
79. Compute $\sin^{-1}(0.654)$ (in radians)
80. Compute $\sin^{-1}(-0.6)$ (in radians)
81. Compute $\cos^{-1}(0.1576)$ (in degrees)
82. Find the general solution to the equation $\sin x = 0.7$ (in radians)
83. A ranger in an observation tower can sight the north end of a lake 15 km away and the south end of the same lake 19 km away. The angle between these two lines of sight is 104°. How long is the lake?
84. Two planes leave an airport at the same time, one flying due west at 500 km/h and the other flying due southeast at 300 km/h. What is the distance between the planes two hours later?
85. A triangular-shaped lot of land has sides of length 130m, 150m and 80m. What are the measures of the angles?
86. Two streets meet at an angle of 52°. If a triangular lot has frontages of 60m and 65m on the two streets, what is the perimeter of the lot?

87. The carnival ferris wheel is 20 feet in diameter and turns at 4 rpm. The bottom of the ferris wheel is 3 feet above the ground. Assume that the height h of a passenger above the ground varies sinusoidally with time t (in seconds).
    a. Find an equation that describes the functional relationship between h and t.
    b. Use the model to predict the height of the passenger above the ground for $t = 3$
    c. When will the passenger be 7 feet from the ground the first time?

88. An airplane flies from Ft. Myers to Sarasota, a distance of 150 miles, and then turns through an angle of 50° and flies to Orlando, a distance of 100 miles.
    a. How far is it from Ft. Myers to Orlando?
    b. Through what angle should the pilot turn at Orlando to return to Ft. Myers?

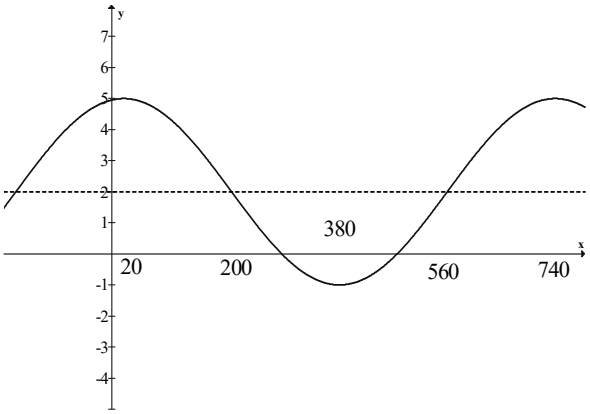


89. Two ships leave port, one traveling in a straight course at 22 miles per hour and the other traveling a straight course at 31 miles per hour. Their courses diverge by 38°. How far apart are they after 3 hours?
90. Calculate the magnitude and direction angle of the vector $\langle -3, 7 \rangle$
91. Calculate the magnitude and direction angle of the vector. $\langle 4, -2 \rangle$
92. Find the components of vector OA where O is the origin of the system of rectangular axes and A is a point 20 units away from the origin at a bearing of 136°
93. Two ropes are attached to a handle on a box. One rope is being pulled with a force of 50 pounds at a 30° angle to the horizontal. The other rope is being pulled with a force of 40 pounds at a 45° angle to the horizontal. Calculate the magnitude and direction angle of the resultant force.
94. Point A is 20 units away from the origin bearing 330° and point B is 30 units away bearing 238°.
    a. Find the components of vector AB
    b. Find the magnitude and bearing.

Answers

1. 30º
2. 35º
3. 5º
4. 38º
5. $\frac{71\pi}{18}$
6. $\frac{3\pi}{4}$
7. −67.5º
8. 40º
9. $\frac{2\sqrt{5}}{5}$
10. $-\frac{5\sqrt{21}}{21}$
11. $\frac{\sqrt{2}}{4}$
12. $-\frac{3}{4}$
13. $-\frac{\sqrt{74}}{5}$
14. All
15. *sin*, *csc*
16. *cos*, *sec*
17. I, II
18. $\sqrt{3}$
19. $\frac{1}{2}$
20. $-\frac{1}{2}$
21. −2
22. $-\sqrt{2}$
23. $\frac{\sqrt{3}}{2}$
24. $-\frac{2\sqrt{3}}{3}$
25. $\frac{1}{2}$
26. 1
27. −2
28. 0
29. 2

30.



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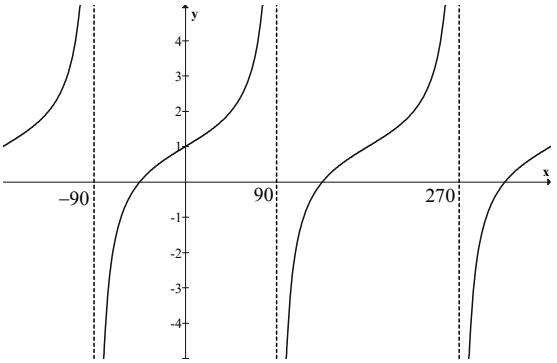

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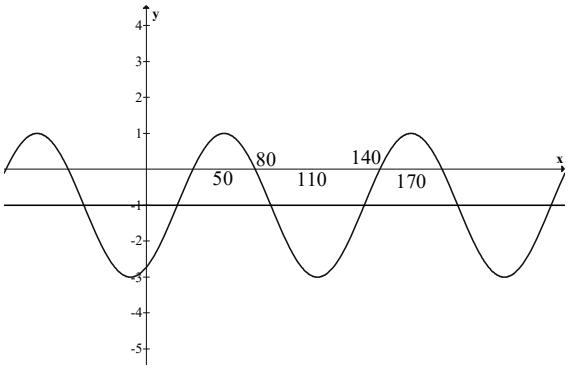

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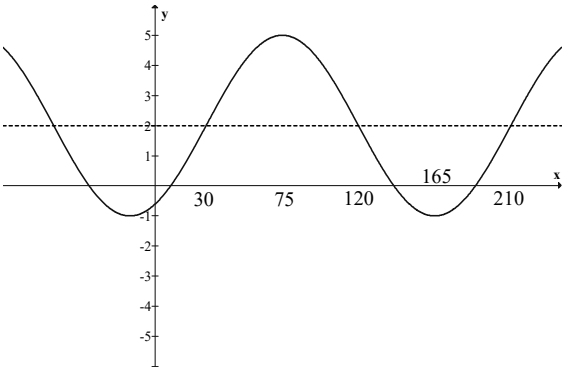

34. 4
35. 15º for *sin*; 60º for *cos*
36. $y = 2 + 3\cos\left(6\left(\theta - 15°\right)\right)$ or $y = 2 + 3sin\left(6\theta\right)$
37. 1440º
38. $y = -2 + 3\cos\left(3\left(x + \frac{\pi}{6}\right)\right)$

    A: 3; B: 120º; C: – 2; D: $\frac{\pi}{6}$
39. $y = 3 + 6\sin(2(x - 100°))$
40. $y = 2 + \sin(3(x - 55°))$
41. $[-1, 5]$
42. 72º
43. up 2
44. 100º right
45. $cot\, x$
46. $cos\, x\, cot\, x$
47. $csc\, x\, cot\, x$
48. $sec\, x$
49. $sin\, x$
50. $csc\, 2x$
51. $sin^2\, x$
52. $\left\{-2\pi, -\frac{5\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi\right\}$
53. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$
54. $\{60, 120, 240, 300\}$
55. $\left\{-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$
56. 60 or $\frac{\pi}{3}$
57. –60 or $-\frac{\pi}{3}$
58. 120 or $\frac{2\pi}{3}$
59. $-\frac{\pi}{6}$
60. $\frac{15}{17}$
61. 30 or $\frac{\pi}{6}$
62. 120 or $\frac{2\pi}{3}$
63. $\frac{\pi}{6}$
64. D: $\left[-\frac{3}{2}, -\frac{1}{2}\right]$; R: $\left[-\pi, 2\pi\right]$
65. D: $\left[\frac{5}{3}, \frac{7}{3}\right]$; R: $\left[-\pi, 3\pi\right]$
66. D: $\left[-5, 1\right]$; R: $\left[2, 5\right]$
67. D: $\left[3, 7\right]$; R: $\left[3, 5\right]$
68. D: $\left(-\infty, \infty\right)$ R: $\left[-\frac{\pi}{3}, \frac{5\pi}{3}\right]$
69. $\frac{\sqrt{6} + \sqrt{2}}{4}$
70. $\sqrt{2} - \sqrt{6}$
71. a. $\frac{1}{9}$

    b. $\sqrt{3}$

    c. $\frac{\sqrt{5} - 2\sqrt{3}}{6}$

    d. $\frac{-\sqrt{5} + 2\sqrt{3}}{6}$
72. 287.04 m
73. $\frac{6547.76\ ft}{5280}$ = 1.24 miles
74. 5.146
75. 28.594
76. 24.437
77. 11.468
78. 46.567º
79. 0.7129
80. -0.6435
81. 80.9324
82. 0.7754 + 2π*n* and 2.3662 + 2π*n*
83. 26.9 km
84. 1486.11 km
85. 32.2º, 87.8º, 60.0º
86. 179.98
87. a. $h = 13 + 10\cos\frac{2\pi}{15}t$

    b. 16.090

    c. 5.286 seconds
88. a. 227.56 mi

    b. 149.7°
89. 57.7
90. 7.616 at 113.199°
91. 4.472 at 333.435°
92. $\langle 13.893, -14.387\rangle$
93. 55.360 at 74.261° to the horizontal
94. a. $\langle -35.441, 1.423\rangle$

    b. 35.470 units bearing 272.299°