SKILLS

5–8 ■ The nth term of a sequence is given. **(a)** Find the first five terms of the sequence. **(b)** What is the common ratio r? **(c)** Graph the terms you found in (a).

5. $a_n = 5(2)^{n-1}$ **6.** $a_n = 3(-4)^{n-1}$

7. $a_n = \frac{5}{2}\left(-\frac{1}{2}\right)^{n-1}$ **8.** $a_n = 3^{n-1}$

9–12 ■ Find the nth term of the geometric sequence with given first term a and common ratio r. What is the fourth term?

9. $a = 3, \quad r = 5$ **10.** $a = -6, \quad r = 3$

11. $a = \frac{5}{2}, \quad r = -\frac{1}{2}$ **12.** $a = \sqrt{3}, \quad r = \sqrt{3}$

13–20 ■ Determine whether the sequence is geometric. If it is geometric, find the common ratio.

13. $2, 4, 8, 16, \ldots$ **14.** $2, 6, 18, 36, \ldots$

15. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$ **16.** $27, -9, 3, -1, \ldots$

17. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ **18.** $e^2, e^4, e^6, e^8, \ldots$

19. $1.0, 1.1, 1.21, 1.331, \ldots$ **20.** $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

21–26 ■ Find the first five terms of the sequence, and determine whether it is geometric. If it is geometric, find the common ratio, and express the nth term of the sequence in the standard form $a_n = ar^{n-1}$.

21. $a_n = 2(3)^n$ **22.** $a_n = 4 + 3^n$

23. $a_n = \dfrac{1}{4^n}$ **24.** $a_n = (-1)^n 2^n$

25. $a_n = \ln(5^{n-1})$ **26.** $a_n = n^n$

27–36 ■ Determine the common ratio, the fifth term, and the nth term of the geometric sequence.

27. $2, 6, 18, 54, \ldots$ **28.** $7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \ldots$

29. $0.3, -0.09, 0.027, -0.0081, \ldots$

30. $1, \sqrt{2}, 2, 2\sqrt{2}, \ldots$

31. $144, -12, 1, -\frac{1}{12}, \ldots$ **32.** $-8, -2, -\frac{1}{2}, -\frac{1}{8}, \ldots$

33. $3, 3^{5/3}, 3^{7/3}, 27, \ldots$ **34.** $t, \dfrac{t^2}{2}, \dfrac{t^3}{4}, \dfrac{t^4}{8}, \ldots$

35. $1, s^{2/7}, s^{4/7}, s^{6/7}, \ldots$ **36.** $5, 5^{c+1}, 5^{2c+1}, 5^{3c+1}, \ldots$

37. The first term of a geometric sequence is 8, and the second term is 4. Find the fifth term.

38. The first term of a geometric sequence is 3, and the third term is $\frac{4}{3}$. Find the fifth term.

39. The common ratio in a geometric sequence is $\frac{2}{5}$, and the fourth term is $\frac{5}{2}$. Find the third term.

40. The common ratio in a geometric sequence is $\frac{3}{2}$, and the fifth term is 1. Find the first three terms.

41. Which term of the geometric sequence 2, 6, 18, . . . is 118,098?

42. The second and fifth terms of a geometric sequence are 10 and 1250, respectively. Is 31,250 a term of this sequence? If so, which term is it?

43–46 ■ Find the partial sum S_n of the geometric sequence that satisfies the given conditions.

43. $a = 5, \quad r = 2, \quad n = 6$ **44.** $a = \frac{2}{3}, \quad r = \frac{1}{3}, \quad n = 4$

45. $a_3 = 28, \quad a_6 = 224, \quad n = 6$

46. $a_2 = 0.12, \quad a_5 = 0.00096, \quad n = 4$

47–50 ■ Find the sum.

47. $1 + 3 + 9 + \cdots + 2187$

48. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{512}$

49. $\displaystyle\sum_{k=0}^{10} 3\left(\tfrac{1}{2}\right)^k$ **50.** $\displaystyle\sum_{j=0}^{5} 7\left(\tfrac{3}{2}\right)^j$

51–62 ■ Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

51. $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \cdots$ **52.** $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \cdots$

53. $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \cdots$ **54.** $\dfrac{2}{5} + \dfrac{4}{25} + \dfrac{8}{125} + \cdots$

55. $1 + \dfrac{3}{2} + \left(\dfrac{3}{2}\right)^2 + \left(\dfrac{3}{2}\right)^3 + \cdots$

56. $\dfrac{1}{3^6} + \dfrac{1}{3^8} + \dfrac{1}{3^{10}} + \dfrac{1}{3^{12}} + \cdots$

57. $3 - \dfrac{3}{2} + \dfrac{3}{4} - \dfrac{3}{8} + \cdots$

58. $1 - 1 + 1 - 1 + \cdots$

59. $3 - 3(1.1) + 3(1.1)^2 - 3(1.1)^3 + \cdots$

60. $-\dfrac{100}{9} + \dfrac{10}{3} - 1 + \dfrac{3}{10} - \cdots$

61. $\dfrac{1}{\sqrt{2}} + \dfrac{1}{2} + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{4} + \cdots$

62. $1 - \sqrt{2} + 2 - 2\sqrt{2} + 4 - \cdots$

63–68 ■ Express the repeating decimal as a fraction.

63. $0.777\ldots$ **64.** $0.2\overline{53}$ **65.** $0.030303\ldots$

66. $2.11\overline{25}$ **67.** $0.\overline{112}$ **68.** $0.123123123\ldots$

69. If the numbers $a_1, a_2, \ldots, a_n$ form a geometric sequence, then $a_2, a_3, \ldots, a_{n-1}$ are **geometric means** between a_1 and a_n. Insert three geometric means between 5 and 80.

70. Find the sum of the first ten terms of the sequence

$$a + b, a^2 + 2b, a^3 + 3b, a^4 + 4b, \ldots$$

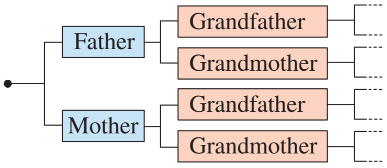
APPLICATIONS

71. Depreciation A construction company purchases a bulldozer for \$160,000. Each year the value of the bulldozer depreciates by 20% of its value in the preceding year. Let V_n be the value of the bulldozer in the nth year. (Let $n = 1$ be the year the bulldozer is purchased.)

(a) Find a formula for V_n.

(b) In what year will the value of the bulldozer be less than \$100,000?

72. Family Tree A person has two parents, four grandparents, eight great-grandparents, and so on. How many ancestors does a person have 15 generations back?

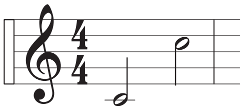


73. Bouncing Ball A ball is dropped from a height of 80 ft. The elasticity of this ball is such that it rebounds three-fourths of the distance it has fallen. How high does the ball rebound on the fifth bounce? Find a formula for how high the ball rebounds on the nth bounce.

74. Bacteria Culture A culture initially has 5000 bacteria, and its size increases by 8% every hour. How many bacteria are present at the end of 5 hours? Find a formula for the number of bacteria present after n hours.

75. Mixing Coolant A truck radiator holds 5 gal and is filled with water. A gallon of water is removed from the radiator and replaced with a gallon of antifreeze; then a gallon of the mixture is removed from the radiator and again replaced by a gallon of antifreeze. This process is repeated indefinitely. How much water remains in the tank after this process is repeated 3 times? 5 times? n times?

76. Musical Frequencies The frequencies of musical notes (measured in cycles per second) form a geometric sequence. Middle C has a frequency of 256, and the C that is an octave higher has a frequency of 512. Find the frequency of C two octaves below middle C.



77. Bouncing Ball A ball is dropped from a height of 9 ft. The elasticity of the ball is such that it always bounces up one-third the distance it has fallen.

(a) Find the total distance the ball has traveled at the instant it hits the ground the fifth time.

(b) Find a formula for the total distance the ball has traveled at the instant it hits the ground the nth time.

78. Geometric Savings Plan A very patient woman wishes to become a billionaire. She decides to follow a simple scheme: She puts aside 1 cent the first day, 2 cents the second day, 4 cents the third day, and so on, doubling the number of cents each day. How much money will she have at the end of 30 days? How many days will it take this woman to realize her wish?

79. St. Ives The following is a well-known children's rhyme:

As I was going to St. Ives,
I met a man with seven wives;
Every wife had seven sacks;
Every sack had seven cats;
Every cat had seven kits;
Kits, cats, sacks, and wives,
How many were going to St. Ives?

Assuming that the entire group is actually going to St. Ives, show that the answer to the question in the rhyme is a partial sum of a geometric sequence, and find the sum.

80. Drug Concentration A certain drug is administered once a day. The concentration of the drug in the patient's bloodstream increases rapidly at first, but each successive dose has less effect than the preceding one. The total amount of the drug (in mg) in the bloodstream after the nth dose is given by

$$\sum_{k=1}^{n} 50\left(\tfrac{1}{2}\right)^{k-1}$$

(a) Find the amount of the drug in the bloodstream after $n = 10$ days.

(b) If the drug is taken on a long-term basis, the amount in the bloodstream is approximated by the infinite series $\sum_{k=1}^{\infty} 50\left(\tfrac{1}{2}\right)^{k-1}$. Find the sum of this series.

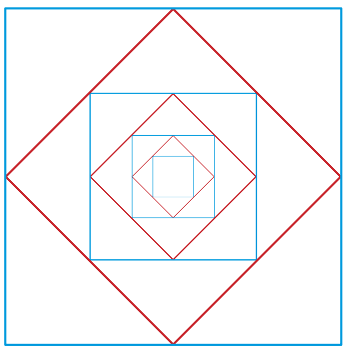
81. Bouncing Ball A certain ball rebounds to half the height from which it is dropped. Use an infinite geometric series to approximate the total distance the ball travels after being dropped from 1 m above the ground until it comes to rest.

82. Bouncing Ball If the ball in Exercise 81 is dropped from a height of 8 ft, then 1 s is required for its first complete bounce—from the instant it first touches the ground until it next touches the ground. Each subsequent complete bounce requires $1/\sqrt{2}$ as long as the preceding complete bounce. Use an infinite geometric series to estimate the time interval from the instant the ball first touches the ground until it stops bouncing.

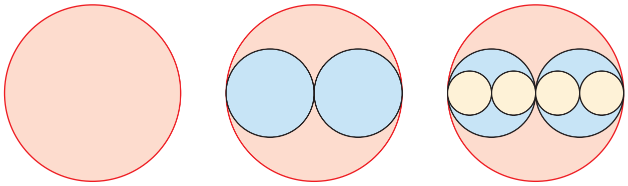
83. Geometry The midpoints of the sides of a square of side 1 are joined to form a new square. This procedure is repeated for each new square. (See the figure.)

(a) Find the sum of the areas of all the squares.

(b) Find the sum of the perimeters of all the squares.

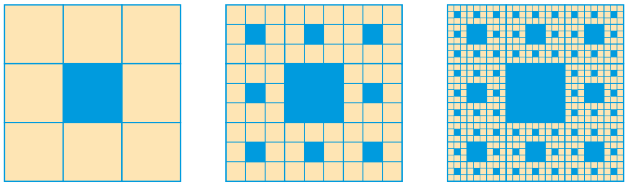


84. Geometry A circular disk of radius R is cut out of paper, as shown in figure (a). Two disks of radius $\frac{1}{2}R$ are cut out of paper and placed on top of the first disk, as in figure (b), and then four disks of radius $\frac{1}{4}R$ are placed on these two disks, as in figure (c). Assuming that this process can be repeated indefinitely, find the total area of all the disks.



(a) (b) (c)

85. Geometry A yellow square of side 1 is divided into nine smaller squares, and the middle square is colored blue as shown in the figure. Each of the smaller yellow squares is in turn divided into nine squares, and each middle square is colored blue. If this process is continued indefinitely, what is the total area that is colored blue?



DISCOVERY ▪ DISCUSSION ▪ WRITING

86. Arithmetic or Geometric? The first four terms of a sequence are given. Determine whether these terms can be the terms of an arithmetic sequence, a geometric sequence, or neither. Find the next term if the sequence is arithmetic or geometric.

(a) $5, -3, 5, -3, \ldots$ **(b)** $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, \ldots$
(c) $\sqrt{3}, 3, 3\sqrt{3}, 9, \ldots$ **(d)** $1, -1, 1, -1, \ldots$
(e) $2, -1, \frac{1}{2}, 2, \ldots$ **(f)** $x - 1, x, x + 1, x + 2, \ldots$
(g) $-3, -\frac{3}{2}, 0, \frac{3}{2}, \ldots$ **(h)** $\sqrt{5}, \sqrt[3]{5}, \sqrt[6]{5}, 1, \ldots$

87. Reciprocals of a Geometric Sequence If $a_1, a_2, a_3, \ldots$ is a geometric sequence with common ratio r, show that the sequence

$$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$$

is also a geometric sequence, and find the common ratio.

88. Logarithms of a Geometric Sequence If $a_1, a_2, a_3, \ldots$ is a geometric sequence with a common ratio $r > 0$ and $a_1 > 0$, show that the sequence

$$\log a_1, \log a_2, \log a_3, \ldots$$

is an arithmetic sequence, and find the common difference.

89. Exponentials of an Arithmetic Sequence If $a_1, a_2, a_3, \ldots$ is an arithmetic sequence with common difference d, show that the sequence

$$10^{a_1}, 10^{a_2}, 10^{a_3}, \ldots$$

is a geometric sequence, and find the common ratio.

DISCOVERY PROJECT **Finding Patterns**

In this project we investigate the process of finding patterns in sequences by using "difference sequences." You can find the project at the book companion website: **www.stewartmath.com**

12.4 Mathematics of Finance

The Amount of an Annuity ▶ The Present Value of an Annuity ▶ Installment Buying

Many financial transactions involve payments that are made at regular intervals. For example, if you deposit \$100 each month in an interest-bearing account, what will the value of your account be at the end of 5 years? If you borrow \$100,000 to buy a house, how much must your monthly payments be in order to pay off the loan in 30 years? Each of these questions involves the sum of a sequence of numbers; we use the results of the preceding section to answer them here.

▼ The Amount of an Annuity

An **annuity** is a sum of money that is paid in regular equal payments. Although the word *annuity* suggests annual (or yearly) payments, they can be made semiannually, quarterly, monthly, or at some other regular interval. Payments are usually made at the end of the payment interval. The **amount of an annuity** is the sum of all the individual payments from the time of the first payment until the last payment is made, together with all the interest. We denote this sum by A_f (the subscript f here is used to denote *final* amount).

EXAMPLE 1 | Calculating the Amount of an Annuity

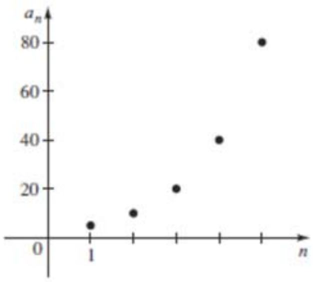
An investor deposits \$400 every December 15 and June 15 for 10 years in an account that earns interest at the rate of 8% per year, compounded semiannually. How much will be in the account immediately after the last payment?

SECTION 12.3 ■ PAGE 805

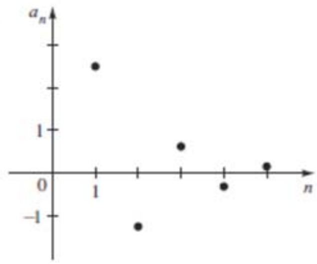
1. ratio **2.** common ratio; 2, 5 **3.** True **4.** **(a)** $a\left(\frac{1-r^n}{1-r}\right)$

(b) geometric; converges, $a/(1-r)$; diverges

5. **(a)** 5, 10, 20, 40, 80
(b) 2
(c)



7. **(a)** $\frac{5}{2}, -\frac{5}{4}, \frac{5}{8}, -\frac{5}{16}, \frac{5}{32}$
(b) $-\frac{1}{2}$
(c)



9. $a_n = 3 \cdot 5^{n-1}, a_4 = 375$ **11.** $a_n = \frac{5}{2}\left(-\frac{1}{2}\right)^{n-1}, a_4 = -\frac{5}{16}$
13. Geometric, 2 **15.** Geometric, $\frac{1}{2}$
17. Not geometric **19.** Geometric, 1.1
21. 6, 18, 54, 162, 486; geometric, common ratio 3; $a_n = 6 \cdot 3^{n-1}$
23. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1024}$; geometric, common ratio $\frac{1}{4}$; $a_n = \frac{1}{4}\left(\frac{1}{4}\right)^{n-1}$
25. 0, ln 5, 2 ln 5, 3 ln 5, 4 ln 5; not geometric
27. 3, $a_5 = 162, a_n = 2 \cdot 3^{n-1}$
29. $-0.3, a_5 = 0.00243, a_n = (0.3)(-0.3)^{n-1}$
31. $-\frac{1}{12}, a_5 = \frac{1}{144}, a_n = 144\left(-\frac{1}{12}\right)^{n-1}$
33. $3^{2/3}, a_5 = 3^{11/3}, a_n = 3^{(2n+1)/3}$
35. $s^{2/7}, a_5 = s^{8/7}, a_n = s^{2(n-1)/7}$
37. $\frac{1}{2}$ **39.** $\frac{25}{4}$ **41.** 11th **43.** 315 **45.** 441
47. 3280
49. $\frac{6141}{1024}$ **51.** $\frac{3}{2}$ **53.** $\frac{3}{4}$ **55.** divergent
57. 2 **59.** divergent **61.** $\sqrt{2} + 1$ **63.** $\frac{7}{9}$ **65.** $\frac{1}{33}$
67. $\frac{112}{999}$ **69.** 10, 20, 40
71. **(a)** $V_n = 160{,}000(0.80)^{n-1}$ **(b)** 4th year
73. 19 ft, $80\left(\frac{3}{4}\right)^n$ **75.** $\frac{64}{25}, \frac{1024}{625}, 5\left(\frac{4}{5}\right)^n$
77. **(a)** $17\frac{8}{9}$ ft **(b)** $18 - \left(\frac{1}{3}\right)^{n-3}$
79. 2801 **81.** 3 m
83. **(a)** 2 **(b)** $8 + 4\sqrt{2}$ **85.** 1