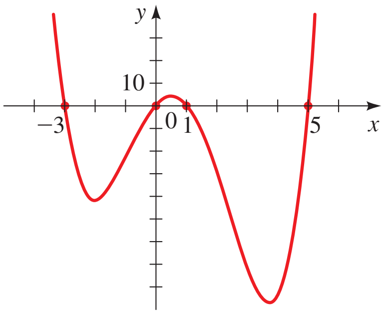


FIGURE 1
$P(x) = (x + 3)x(x - 1)(x - 5)$ has zeros -3, 0, 1, and 5.

Let

$$P(x) = (x + 3)(x - 0)(x - 1)(x - 5)$$
$$= x^4 - 3x^3 - 13x^2 + 15x$$

Since $P(x)$ is of degree 4, it is a solution of the problem. Any other solution of the problem must be a constant multiple of $P(x)$, since only multiplication by a constant does not change the degree.

NOW TRY EXERCISE **59**

The polynomial P of Example 6 is graphed in Figure 1. Note that the zeros of P correspond to the x-intercepts of the graph.

3.3 EXERCISES

CONCEPTS

1. If we divide the polynomial P by the factor $x - c$ and we obtain the equation $P(x) = (x - c)Q(x) + R(x)$, then we say that $x - c$ is the divisor, $Q(x)$ is the ________, and $R(x)$ is the ________.

2. **(a)** If we divide the polynomial $P(x)$ by the factor $x - c$ and we obtain a remainder of 0, then we know that c is a ________ of P.

(b) If we divide the polynomial $P(x)$ by the factor $x - c$ and we obtain a remainder of k, then we know that $P(c) =$ ________.

SKILLS

3–8 ■ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express P in the form $P(x) = D(x) \cdot Q(x) + R(x)$.

3. $P(x) = 3x^2 + 5x - 4, \quad D(x) = x + 3$

4. $P(x) = x^3 + 4x^2 - 6x + 1, \quad D(x) = x - 1$

5. $P(x) = 2x^3 - 3x^2 - 2x, \quad D(x) = 2x - 3$

6. $P(x) = 4x^3 + 7x + 9, \quad D(x) = 2x + 1$

7. $P(x) = x^4 - x^3 + 4x + 2, \quad D(x) = x^2 + 3$

8. $P(x) = 2x^5 + 4x^4 - 4x^3 - x - 3, \quad D(x) = x^2 - 2$

9–14 ■ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

9. $P(x) = x^2 + 4x - 8, \quad D(x) = x + 3$

10. $P(x) = x^3 + 6x + 5, \quad D(x) = x - 4$

11. $P(x) = 4x^2 - 3x - 7, \quad D(x) = 2x - 1$

12. $P(x) = 6x^3 + x^2 - 12x + 5, \quad D(x) = 3x - 4$

13. $P(x) = 2x^4 - x^3 + 9x^2, \quad D(x) = x^2 + 4$

14. $P(x) = x^5 + x^4 - 2x^3 + x + 1, \quad D(x) = x^2 + x - 1$

15–24 ■ Find the quotient and remainder using long division.

15. $\dfrac{x^2 - 6x - 8}{x - 4}$

16. $\dfrac{x^3 - x^2 - 2x + 6}{x - 2}$

17. $\dfrac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$

18. $\dfrac{x^3 + 3x^2 + 4x + 3}{3x + 6}$

19. $\dfrac{x^3 + 6x + 3}{x^2 - 2x + 2}$

20. $\dfrac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$

21. $\dfrac{6x^3 + 2x^2 + 22x}{2x^2 + 5}$

22. $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$

23. $\dfrac{x^6 + x^4 + x^2 + 1}{x^2 + 1}$

24. $\dfrac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$

25–38 ■ Find the quotient and remainder using synthetic division.

25. $\dfrac{x^2 - 5x + 4}{x - 3}$

26. $\dfrac{x^2 - 5x + 4}{x - 1}$

27. $\dfrac{3x^2 + 5x}{x - 6}$

28. $\dfrac{4x^2 - 3}{x + 5}$

29. $\dfrac{x^3 + 2x^2 + 2x + 1}{x + 2}$

30. $\dfrac{3x^3 - 12x^2 - 9x + 1}{x - 5}$

31. $\dfrac{x^3 - 8x + 2}{x + 3}$

32. $\dfrac{x^4 - x^3 + x^2 - x + 2}{x - 2}$

33. $\dfrac{x^5 + 3x^3 - 6}{x - 1}$

34. $\dfrac{x^3 - 9x^2 + 27x - 27}{x - 3}$

35. $\dfrac{2x^3 + 3x^2 - 2x + 1}{x - \frac{1}{2}}$

36. $\dfrac{6x^4 + 10x^3 + 5x^2 + x + 1}{x + \frac{2}{3}}$

37. $\dfrac{x^3 - 27}{x - 3}$ **38.** $\dfrac{x^4 - 16}{x + 2}$

39–51 ■ Use synthetic division and the Remainder Theorem to evaluate $P(c)$.

39. $P(x) = 4x^2 + 12x + 5, \quad c = -1$

40. $P(x) = 2x^2 + 9x + 1, \quad c = \frac{1}{2}$

41. $P(x) = x^3 + 3x^2 - 7x + 6, \quad c = 2$

42. $P(x) = x^3 - x^2 + x + 5, \quad c = -1$

43. $P(x) = x^3 + 2x^2 - 7, \quad c = -2$

44. $P(x) = 2x^3 - 21x^2 + 9x - 200, \quad c = 11$

45. $P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14, \quad c = -7$

46. $P(x) = 6x^5 + 10x^3 + x + 1, \quad c = -2$

47. $P(x) = x^7 - 3x^2 - 1, \quad c = 3$

48. $P(x) = -2x^6 + 7x^5 + 40x^4 - 7x^2 + 10x + 112, \quad c = -3$

49. $P(x) = 3x^3 + 4x^2 - 2x + 1, \quad c = \frac{2}{3}$

50. $P(x) = x^3 - x + 1, \quad c = \frac{1}{4}$

51. $P(x) = x^3 + 2x^2 - 3x - 8, \quad c = 0.1$

52. Let

$$P(x) = 6x^7 - 40x^6 + 16x^5 - 200x^4 - 60x^3 - 69x^2 + 13x - 139$$

Calculate $P(7)$ by **(a)** using synthetic division and **(b)** substituting $x = 7$ into the polynomial and evaluating directly.

53–56 ■ Use the Factor Theorem to show that $x - c$ is a factor of $P(x)$ for the given value(s) of c.

53. $P(x) = x^3 - 3x^2 + 3x - 1, \quad c = 1$

54. $P(x) = x^3 + 2x^2 - 3x - 10, \quad c = 2$

55. $P(x) = 2x^3 + 7x^2 + 6x - 5, \quad c = \frac{1}{2}$

56. $P(x) = x^4 + 3x^3 - 16x^2 - 27x + 63, \quad c = 3, -3$

57–58 ■ Show that the given value(s) of c are zeros of $P(x)$, and find all other zeros of $P(x)$.

57. $P(x) = x^3 - x^2 - 11x + 15, \quad c = 3$

58. $P(x) = 3x^4 - x^3 - 21x^2 - 11x + 6, \quad c = \frac{1}{3}, -2$

59–62 ■ Find a polynomial of the specified degree that has the given zeros.

59. Degree 3; zeros $-1, 1, 3$

60. Degree 4; zeros $-2, 0, 2, 4$

61. Degree 4; zeros $-1, 1, 3, 5$

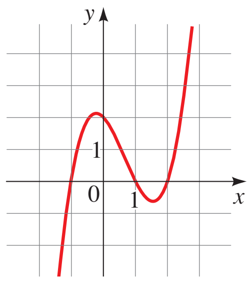
62. Degree 5; zeros $-2, -1, 0, 1, 2$

63. Find a polynomial of degree 3 that has zeros 1, -2, and 3 and in which the coefficient of x^2 is 3.

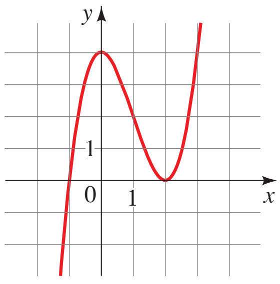
64. Find a polynomial of degree 4 that has integer coefficients and zeros 1, -1, 2, and $\frac{1}{2}$.

65–68 ■ Find the polynomial of the specified degree whose graph is shown.

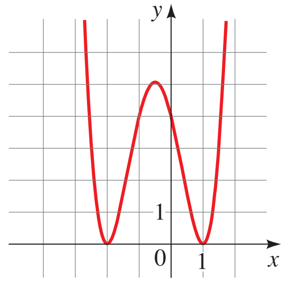
65. Degree 3



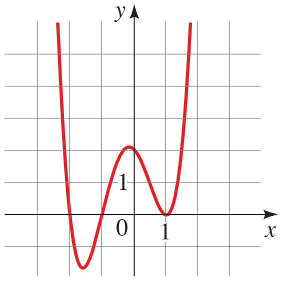
66. Degree 3



67. Degree 4



68. Degree 4



DISCOVERY ■ DISCUSSION ■ WRITING

69. Impossible Division? Suppose you were asked to solve the following two problems on a test:

A. Find the remainder when $6x^{1000} - 17x^{562} + 12x + 26$ is divided by $x + 1$.

B. Is $x - 1$ a factor of $x^{567} - 3x^{400} + x^9 + 2$?

Obviously, it's impossible to solve these problems by dividing, because the polynomials are of such large degree. Use one or more of the theorems in this section to solve these problems *without* actually dividing.

70. Nested Form of a Polynomial Expand Q to prove that the polynomials P and Q are the same.

$$P(x) = 3x^4 - 5x^3 + x^2 - 3x + 5$$

$$Q(x) = (((3x - 5)x + 1)x - 3)x + 5$$

Try to evaluate $P(2)$ and $Q(2)$ in your head, using the forms given. Which is easier? Now write the polynomial $R(x) = x^5 - 2x^4 + 3x^3 - 2x^2 + 3x + 4$ in "nested" form, like the polynomial Q. Use the nested form to find $R(3)$ in your head.

Do you see how calculating with the nested form follows the same arithmetic steps as calculating the value of a polynomial using synthetic division?

SECTION 3.3 ■ PAGE 251

1. quotient, remainder **2. (a)** factor **(b)** k
3. $(x + 3)(3x - 4) + 8$ **5.** $(2x - 3)(x^2 - 1) - 3$
7. $(x^2 + 3)(x^2 - x - 3) + (7x + 11)$
9. $x + 1 + \dfrac{-11}{x + 3}$ **11.** $2x - \dfrac{1}{2} + \dfrac{-\frac{15}{2}}{2x - 1}$
13. $2x^2 - x + 1 + \dfrac{4x - 4}{x^2 + 4}$

In answers 15–37 the first polynomial given is the quotient, and the second is the remainder.

15. $x - 2, -16$ **17.** $2x^2 - 1, -2$ **19.** $x + 2, 8x - 1$
21. $3x + 1, 7x - 5$ **23.** $x^4 + 1, 0$ **25.** $x - 2, -2$
27. $3x + 23, 138$ **29.** $x^2 + 2, -3$ **31.** $x^2 - 3x + 1, -1$
33. $x^4 + x^3 + 4x^2 + 4x + 4, -2$ **35.** $2x^2 + 4x, 1$
37. $x^2 + 3x + 9, 0$ **39.** -3 **41.** 12 **43.** -7 **45.** -483
47. 2159 **49.** $\frac{7}{3}$ **51.** -8.279 **57.** $-1 \pm \sqrt{6}$
59. $x^3 - 3x^2 - x + 3$ **61.** $x^4 - 8x^3 + 14x^2 + 8x - 15$
63. $-\frac{3}{2}x^3 + 3x^2 + \frac{15}{2}x - 9$ **65.** $(x + 1)(x - 1)(x - 2)$
67. $(x + 2)^2(x - 1)^2$