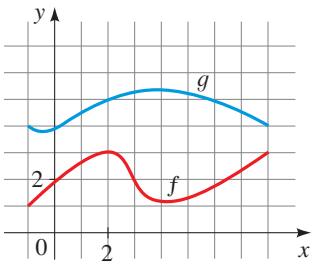
2.6 EXERCISES

CONCEPTS

1. From the graphs of f and g in the figure, we find

$(f + g)(2) =$ _____ $(f - g)(2) =$ _____

$(fg)(2) =$ _____ $\left(\frac{f}{g}\right)(2) =$ _____



2. By definition, $f \circ g(x) =$ _____. So if $g(2) = 5$ and $f(5) = 12$, then $f \circ g(2) =$ _____.

3. If the rule of the function f is "add one" and the rule of the function g is "multiply by 2," then the rule of $f \circ g$ is

"_____,"

and the rule of $g \circ f$ is

"_____."

4. We can express the functions in Exercise 3 algebraically as

$f(x) =$ _____ $g(x) =$ _____

$f \circ g(x) =$ _____ $g \circ f(x) =$ _____

SKILLS

5–10 ■ Find $f + g$, $f - g$, fg, and f/g and their domains.

5. $f(x) = x - 3, \quad g(x) = x^2$

6. $f(x) = x^2 + 2x, \quad g(x) = 3x^2 - 1$

7. $f(x) = \sqrt{4 - x^2}, \quad g(x) = \sqrt{1 + x}$

8. $f(x) = \sqrt{9 - x^2}, \quad g(x) = \sqrt{x^2 - 4}$

9. $f(x) = \dfrac{2}{x}, \quad g(x) = \dfrac{4}{x + 4}$

10. $f(x) = \dfrac{2}{x + 1}, \quad g(x) = \dfrac{x}{x + 1}$

11–14 ■ Find the domain of the function.

11. $f(x) = \sqrt{x} + \sqrt{1 - x}$

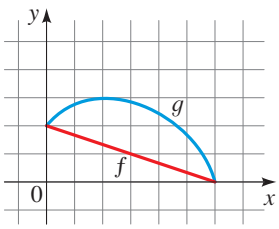
12. $g(x) = \sqrt{x + 1} - \dfrac{1}{x}$

13. $h(x) = (x - 3)^{-1/4}$

14. $k(x) = \dfrac{\sqrt{x + 3}}{x - 1}$

15–16 ■ Use graphical addition to sketch the graph of $f + g$.

15.



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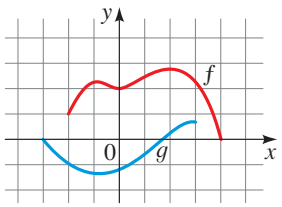


17–20 ■ Draw the graphs of f, g, and $f + g$ on a common screen to illustrate graphical addition.

17. $f(x) = \sqrt{1 + x}, \quad g(x) = \sqrt{1 - x}$

18. $f(x) = x^2, \quad g(x) = \sqrt{x}$

19. $f(x) = x^2, \quad g(x) = \frac{1}{3}x^3$

20. $f(x) = \sqrt[4]{1 - x}, \quad g(x) = \sqrt{1 - \dfrac{x^2}{9}}$

21–26 ■ Use $f(x) = 3x - 5$ and $g(x) = 2 - x^2$ to evaluate the expression.

21. **(a)** $f(g(0))$ **(b)** $g(f(0))$

22. **(a)** $f(f(4))$ **(b)** $g(g(3))$

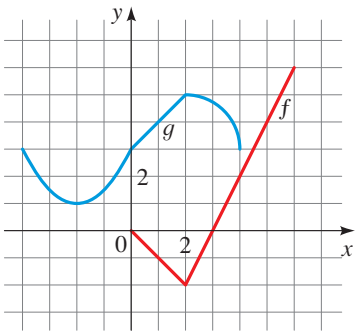
23. **(a)** $(f \circ g)(-2)$ **(b)** $(g \circ f)(-2)$

24. **(a)** $(f \circ f)(-1)$ **(b)** $(g \circ g)(2)$

25. **(a)** $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$

26. **(a)** $(f \circ f)(x)$ **(b)** $(g \circ g)(x)$

27–32 ■ Use the given graphs of f and g to evaluate the expression.



27. $f(g(2))$

28. $g(f(0))$

29. $(g \circ f)(4)$

30. $(f \circ g)(0)$

31. $(g \circ g)(-2)$

32. $(f \circ f)(4)$

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33–44 ■ Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

33. $f(x) = 2x + 3, \quad g(x) = 4x - 1$

34. $f(x) = 6x - 5, \quad g(x) = \dfrac{x}{2}$

35. $f(x) = x^2, \quad g(x) = x + 1$

36. $f(x) = x^3 + 2, \quad g(x) = \sqrt[3]{x}$

37. $f(x) = \dfrac{1}{x}, \quad g(x) = 2x + 4$

38. $f(x) = x^2, \quad g(x) = \sqrt{x - 3}$

39. $f(x) = |x|, \quad g(x) = 2x + 3$

40. $f(x) = x - 4, \quad g(x) = |x + 4|$

41. $f(x) = \dfrac{x}{x + 1}, \quad g(x) = 2x - 1$

42. $f(x) = \dfrac{1}{\sqrt{x}}, \quad g(x) = x^2 - 4x$

43. $f(x) = \dfrac{x}{x + 1}, \quad g(x) = \dfrac{1}{x}$

44. $f(x) = \dfrac{2}{x}, \quad g(x) = \dfrac{x}{x + 2}$

45–48 ■ Find $f \circ g \circ h$.

45. $f(x) = x - 1, \quad g(x) = \sqrt{x}, \quad h(x) = x - 1$

46. $f(x) = \dfrac{1}{x}, \quad g(x) = x^3, \quad h(x) = x^2 + 2$

47. $f(x) = x^4 + 1, \quad g(x) = x - 5, \quad h(x) = \sqrt{x}$

48. $f(x) = \sqrt{x}, \quad g(x) = \dfrac{x}{x - 1}, \quad h(x) = \sqrt[3]{x}$

49–54 ■ Express the function in the form $f \circ g$.

49. $F(x) = (x - 9)^5$

50. $F(x) = \sqrt{x} + 1$

51. $G(x) = \dfrac{x^2}{x^2 + 4}$

52. $G(x) = \dfrac{1}{x + 3}$

53. $H(x) = |1 - x^3|$

54. $H(x) = \sqrt{1 + \sqrt{x}}$

55–58 ■ Express the function in the form $f \circ g \circ h$.

55. $F(x) = \dfrac{1}{x^2 + 1}$

56. $F(x) = \sqrt[3]{\sqrt{x} - 1}$

57. $G(x) = (4 + \sqrt[3]{x})^9$

58. $G(x) = \dfrac{2}{(3 + \sqrt{x})^2}$

APPLICATIONS

59–60 ■ **Revenue, Cost, and Profit** A print shop makes bumper stickers for election campaigns. If x stickers are ordered (where $x < 10{,}000$), then the price per bumper sticker is $0.15 - 0.000002x$ dollars, and the total cost of producing the order is $0.095x - 0.0000005x^2$ dollars.

59. Use the fact that

$$\text{revenue} = \text{price per item} \times \text{number of items sold}$$

to express $R(x)$, the revenue from an order of x stickers, as a product of two functions of x.

60. Use the fact that

$$\text{profit} = \text{revenue} - \text{cost}$$

to express $P(x)$, the profit on an order of x stickers, as a difference of two functions of x.

61. Area of a Ripple A stone is dropped in a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.

(a) Find a function g that models the radius as a function of time.

(b) Find a function f that models the area of the circle as a function of the radius.

(c) Find $f \circ g$. What does this function represent?

62. Inflating a Balloon A spherical balloon is being inflated. The radius of the balloon is increasing at the rate of 1 cm/s.

(a) Find a function f that models the radius as a function of time.

(b) Find a function g that models the volume as a function of the radius.

(c) Find $g \circ f$. What does this function represent?

63. Area of a Balloon A spherical weather balloon is being inflated. The radius of the balloon is increasing at the rate of 2 cm/s. Express the surface area of the balloon as a function of time t (in seconds).

64. Multiple Discounts You have a $50 coupon from the manufacturer good for the purchase of a cell phone. The store where you are purchasing your cell phone is offering a 20% discount on all cell phones. Let x represent the regular price of the cell phone.

(a) Suppose only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the regular price x.

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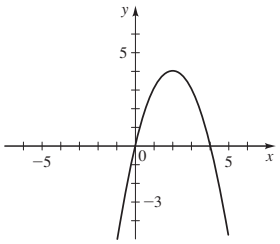
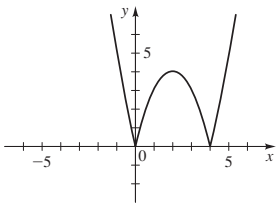
85. To obtain the graph of g, reflect in the x-axis the part of the graph of f that is below the x-axis.

87. (a)



(b)



89. (a) Shift upward 4 units, shrink vertically by a factor of 0.01
(b) Shift to the right 10 units; $g(t) = 4 + 0.01(t - 10)^2$

SECTION 2.6 ■ PAGE 196

1. 8, −2, 15, $\frac{3}{5}$ **2.** $f(g(x))$, 12 **3.** Multiply by 2, then add 1; Add 1, then multiply by 2 **4.** $x + 1, 2x, 2x + 1, 2(x + 1)$

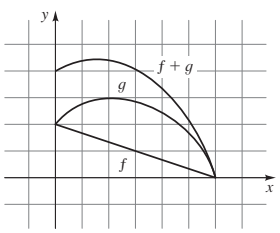
5. $(f + g)(x) = x^2 + x - 3, (-\infty, \infty)$;
$(f - g)(x) = -x^2 + x - 3, (-\infty, \infty)$;
$(fg)(x) = x^3 - 3x^2, (-\infty, \infty)$;
$\left(\frac{f}{g}\right)(x) = \frac{x - 3}{x^2}, (-\infty, 0) \cup (0, \infty)$

7. $(f + g)(x) = \sqrt{4 - x^2} + \sqrt{1 + x}, [-1, 2]$;
$(f - g)(x) = \sqrt{4 - x^2} - \sqrt{1 + x}, [-1, 2]$;
$(fg)(x) = \sqrt{-x^3 - x^2 + 4x + 4}, [-1, 2]$;
$\left(\frac{f}{g}\right)(x) = \sqrt{\frac{4 - x^2}{1 + x}}, (-1, 2]$

9. $(f + g)(x) = \frac{6x + 8}{x^2 + 4x}, x \neq -4, x \neq 0$;
$(f - g)(x) = \frac{-2x + 8}{x^2 + 4x}, x \neq -4, x \neq 0$;
$(fg)(x) = \frac{8}{x^2 + 4x}, x \neq -4, x \neq 0$;
$\left(\frac{f}{g}\right)(x) = \frac{x + 4}{2x}, x \neq -4, x \neq 0$

11. $[0, 1]$ **13.** $(3, \infty)$

15.



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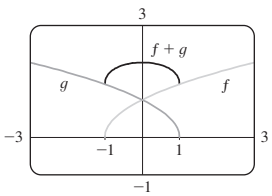

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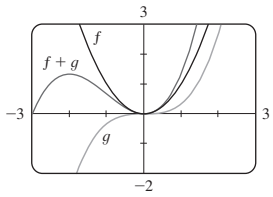

21. (a) 1 **(b)** −23 **23. (a)** −11 **(b)** −119
25. (a) $-3x^2 + 1$ **(b)** $-9x^2 + 30x - 23$
27. 4 **29.** 5 **31.** 4
33. $(f \circ g)(x) = 8x + 1, (-\infty, \infty)$;
$(g \circ f)(x) = 8x + 11, (-\infty, \infty)$; $(f \circ f)(x) = 4x + 9, (-\infty, \infty)$;
$(g \circ g)(x) = 16x - 5, (-\infty, \infty)$
35. $(f \circ g)(x) = (x + 1)^2, (-\infty, \infty)$;
$(g \circ f)(x) = x^2 + 1, (-\infty, \infty)$; $(f \circ f)(x) = x^4, (-\infty, \infty)$;
$(g \circ g)(x) = x + 2, (-\infty, \infty)$
37. $(f \circ g)(x) = \frac{1}{2x + 4}, x \neq -2$; $(g \circ f)(x) = \frac{2}{x} + 4, x \neq 0$;
$(f \circ f)(x) = x, x \neq 0$, $(g \circ g)(x) = 4x + 12, (-\infty, \infty)$
39. $(f \circ g)(x) = |2x + 3|, (-\infty, \infty)$;
$(g \circ f)(x) = 2|x| + 3, (-\infty, \infty)$; $(f \circ f)(x) = |x|, (-\infty, \infty)$;
$(g \circ g)(x) = 4x + 9, (-\infty, \infty)$
41. $(f \circ g)(x) = \frac{2x - 1}{2x}, x \neq 0$; $(g \circ f)(x) = \frac{2x}{x + 1} - 1, x \neq -1$;
$(f \circ f)(x) = \frac{x}{2x + 1}, x \neq -1, x \neq -\frac{1}{2}$;
$(g \circ g)(x) = 4x - 3, (-\infty, \infty)$
43. $(f \circ g)(x) = \frac{1}{x + 1}, x \neq -1, x \neq 0$; $(g \circ f)(x) = \frac{x + 1}{x}$,
$x \neq -1, x \neq 0$; $(f \circ f)(x) = \frac{x}{2x + 1}, x \neq -1, x \neq -\frac{1}{2}$;
$(g \circ g)(x) = x, x \neq 0$
45. $(f \circ g \circ h)(x) = \sqrt{x - 1} - 1$
47. $(f \circ g \circ h)(x) = (\sqrt{x} - 5)^4 + 1$
49. $g(x) = x - 9, f(x) = x^5$
51. $g(x) = x^2, f(x) = x/(x + 4)$
53. $g(x) = 1 - x^3, f(x) = |x|$
55. $h(x) = x^2, g(x) = x + 1, f(x) = 1/x$
57. $h(x) = \sqrt[3]{x}, g(x) = 4 + x, f(x) = x^9$
59. $R(x) = 0.15x - 0.000002x^2$
61. (a) $g(t) = 60t$ **(b)** $f(r) = \pi r^2$ **(c)** $(f \circ g)(t) = 3600\pi t^2$
63. $A(t) = 16\pi t^2$ **65. (a)** $f(x) = 0.9x$ **(b)** $g(x) = x - 100$
(c) $(f \circ g)(x) = 0.9x - 90, (g \circ f)(x) = 0.9x - 100, f \circ g$:
first rebate, then discount, $g \circ f$:
first discount, then rebate, $g \circ f$ is the better deal

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