1.7 EXERCISES

CONCEPTS

1. Fill in the blank with an appropriate inequality sign.

(a) If $x < 5$, then $x - 3$ ______ 2.

(b) If $x \le 5$, then $3x$ ______ 15.

(c) If $x \ge 2$, then $-3x$ ______ -6.

(d) If $x < -2$, then $-x$ ______ 2.

2. *True or false*?

(a) If $x(x + 1) > 0$, then x and $x + 1$ are either both positive or both negative.

(b) If $x(x + 1) > 5$, then x and $x + 1$ are each greater than 5.

3. (a) The solution of the inequality $|x| \le 3$ is the interval ______.

(b) The solution of the inequality $|x| \ge 3$ is a union of two intervals ______ $\cup$ ______.

4. (a) The set of all points on the real line whose distance from zero is less than 3 can be described by the absolute value inequality $|x|$______.

(b) The set of all points on the real line whose distance from zero is greater than 3 can be described by the absolute value inequality $|x|$ ______.

SKILLS

5–10 ■ Let $S = \{-2, -1, 0, \frac{1}{2}, 1, \sqrt{2}, 2, 4\}$. Determine which elements of S satisfy the inequality.

5. $3 - 2x \le \frac{1}{2}$

6. $2x - 1 \ge x$

7. $1 < 2x - 4 \le 7$

8. $-2 \le 3 - x < 2$

9. $\dfrac{1}{x} \le \dfrac{1}{2}$

10. $x^2 + 2 < 4$

11–34 ■ Solve the linear inequality. Express the solution using interval notation and graph the solution set.

11. $2x \le 7$

12. $-4x \ge 10$

13. $2x - 5 > 3$

14. $3x + 11 < 5$

15. $7 - x \ge 5$

16. $5 - 3x \le -16$

17. $2x + 1 < 0$

18. $0 < 5 - 2x$

19. $3x + 11 \le 6x + 8$

20. $6 - x \ge 2x + 9$

21. $\frac{1}{2}x - \frac{2}{3} > 2$

22. $\frac{2}{5}x + 1 < \frac{1}{5} - 2x$

23. $\frac{1}{3}x + 2 < \frac{1}{6}x - 1$

24. $\frac{2}{3} - \frac{1}{2}x \ge \frac{1}{6} + x$

25. $4 - 3x \le -(1 + 8x)$

26. $2(7x - 3) \le 12x + 16$

27. $2 \le x + 5 < 4$

28. $5 \le 3x - 4 \le 14$

29. $-1 < 2x - 5 < 7$

30. $1 < 3x + 4 \le 16$

31. $-2 < 8 - 2x \le -1$

32. $-3 \le 3x + 7 \le \frac{1}{2}$

33. $\dfrac{1}{6} < \dfrac{2x - 13}{12} \le \dfrac{2}{3}$

34. $-\dfrac{1}{2} \le \dfrac{4 - 3x}{5} \le \dfrac{1}{4}$

35–72 ■ Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

35. $(x + 2)(x - 3) < 0$

36. $(x - 5)(x + 4) \ge 0$

37. $x(2x + 7) \ge 0$

38. $x(2 - 3x) \le 0$

39. $x^2 - 3x - 18 \le 0$

40. $x^2 + 5x + 6 > 0$

41. $2x^2 + x \ge 1$

42. $x^2 < x + 2$

43. $3x^2 - 3x < 2x^2 + 4$

44. $5x^2 + 3x \ge 3x^2 + 2$

45. $x^2 > 3(x + 6)$

46. $x^2 + 2x > 3$

47. $x^2 < 4$

48. $x^2 \ge 9$

49. $(x + 2)(x - 1)(x - 3) \le 0$

50. $(x - 5)(x - 2)(x + 1) > 0$

51. $(x - 4)(x + 2)^2 < 0$

52. $(x + 3)^2(x + 1) > 0$

53. $(x - 2)^2(x - 3)(x + 1) \le 0$

54. $x^2(x^2 - 1) \ge 0$

55. $x^3 - 4x > 0$

56. $16x \le x^3$

57. $\dfrac{x - 3}{x + 1} \ge 0$

58. $\dfrac{2x + 6}{x - 2} < 0$

59. $\dfrac{4x}{2x + 3} > 2$

60. $-2 < \dfrac{x + 1}{x - 3}$

61. $\dfrac{2x + 1}{x - 5} \le 3$

62. $\dfrac{3 + x}{3 - x} \ge 1$

63. $\dfrac{4}{x} < x$

64. $\dfrac{x}{x + 1} > 3x$

65. $1 + \dfrac{2}{x + 1} \le \dfrac{2}{x}$

66. $\dfrac{3}{x - 1} - \dfrac{4}{x} \ge 1$

67. $\dfrac{6}{x - 1} - \dfrac{6}{x} \ge 1$

68. $\dfrac{x}{2} \ge \dfrac{5}{x + 1} + 4$

69. $\dfrac{x + 2}{x + 3} < \dfrac{x - 1}{x - 2}$

70. $\dfrac{1}{x + 1} + \dfrac{1}{x + 2} \le 0$

71. $x^4 > x^2$

72. $x^5 > x^2$

73–88 ■ Solve the absolute value inequality. Express the answer using interval notation and graph the solution set.

73. $|x| \le 4$

74. $|3x| < 15$

75. $|2x| > 7$

76. $\frac{1}{2}|x| \ge 1$

77. $|x - 5| \le 3$

78. $|x + 1| \ge 1$

79. $|2x - 3| \le 0.4$

80. $|5x - 2| < 6$

81. $|3x - 2| \ge 5$

82. $|8x + 3| > 12$

83. $\left|\dfrac{x - 2}{3}\right| < 2$

84. $\left|\dfrac{x + 1}{2}\right| \ge 4$

85. $|x + 6| < 0.001$

86. $3 - |2x + 4| \le 1$

87. $8 - |2x - 1| \ge 6$

88. $7|x + 2| + 5 > 4$

89–92 ■ A phrase describing a set of real numbers is given. Express the phrase as an inequality involving an absolute value.

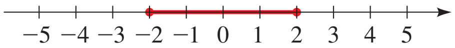
89. All real numbers x less than 3 units from 0

90. All real numbers x more than 2 units from 0

91. All real numbers x at least 5 units from 7

92. All real numbers x at most 4 units from 2

93–98 ■ A set of real numbers is graphed. Find an inequality involving an absolute value that describes the set.

93.


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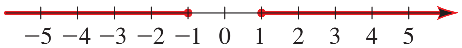

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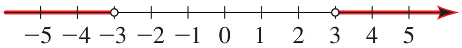

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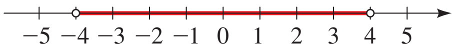

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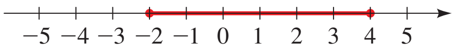

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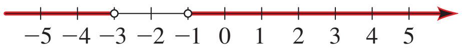


99–102 ■ Determine the values of the variable for which the expression is defined as a real number.

99. $\sqrt{16 - 9x^2}$ **100.** $\sqrt{3x^2 - 5x + 2}$

101. $\left(\dfrac{1}{x^2 - 5x - 14}\right)^{1/2}$ **102.** $\sqrt[4]{\dfrac{1 - x}{2 + x}}$

103. Solve the inequality for x, assuming that a, b, and c are positive constants.

(a) $a(bx - c) \geq bc$ **(b)** $a \leq bx + c < 2a$

104. Suppose that a, b, c, and d are positive numbers such that

$$\frac{a}{b} < \frac{c}{d}$$

Show that $\dfrac{a}{b} < \dfrac{a + c}{b + d} < \dfrac{c}{d}$.

APPLICATIONS

105. Temperature Scales Use the relationship between C and F given in Example 9 to find the interval on the Fahrenheit scale corresponding to the temperature range $20 \leq C \leq 30$.

106. Temperature Scales What interval on the Celsius scale corresponds to the temperature range $50 \leq F \leq 95$?

107. Car Rental Cost A car rental company offers two plans for renting a car.

Plan A: $30 per day and 10¢ per mile

Plan B: $50 per day with free unlimited mileage

For what range of miles will Plan B save you money?

108. Long-Distance Cost A telephone company offers two long-distance plans.

Plan A: $25 per month and 5¢ per minute

Plan B: $5 per month and 12¢ per minute

For how many minutes of long-distance calls would Plan B be financially advantageous?

109. Driving Cost It is estimated that the annual cost of driving a certain new car is given by the formula

$$C = 0.35m + 2200$$

where m represents the number of miles driven per year and C is the cost in dollars. Jane has purchased such a car and decides to budget between $6400 and $7100 for next year's driving costs. What is the corresponding range of miles that she can drive her new car?

110. Air Temperature As dry air moves upward, it expands and, in so doing, cools at a rate of about 1°C for each 100-meter rise, up to about 12 km.

(a) If the ground temperature is 20°C, write a formula for the temperature at height h.

(b) What range of temperatures can be expected if a plane takes off and reaches a maximum height of 5 km?

111. Airline Ticket Price A charter airline finds that on its Saturday flights from Philadelphia to London all 120 seats will be sold if the ticket price is $200. However, for each $3 increase in ticket price, the number of seats sold decreases by one.

(a) Find a formula for the number of seats sold if the ticket price is P dollars.

(b) Over a certain period the number of seats sold for this flight ranged between 90 and 115. What was the corresponding range of ticket prices?

112. Accuracy of a Scale A coffee merchant sells a customer 3 lb of Hawaiian Kona at $6.50 per pound. The merchant's scale is accurate to within ±0.03 lb. By how much could the customer have been overcharged or undercharged because of possible inaccuracy in the scale?

113. Gravity The gravitational force F exerted by the earth on an object having a mass of 100 kg is given by the equation

$$F = \frac{4{,}000{,}000}{d^2}$$

where d is the distance (in km) of the object from the center of the earth, and the force F is measured in newtons (N). For what distances will the gravitational force exerted by the earth on this object be between 0.0004 N and 0.01 N?

114. Bonfire Temperature In the vicinity of a bonfire the temperature T in °C at a distance of x meters from the center of the fire was given by

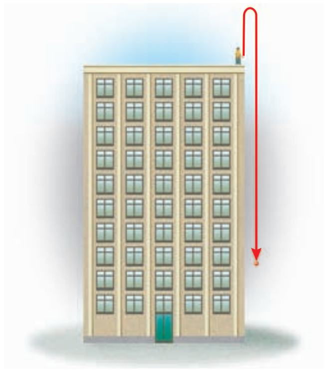
$$T = \frac{600{,}000}{x^2 + 300}$$

At what range of distances from the fire's center was the temperature less than 500°C?

115. Falling Ball Using calculus, it can be shown that if a ball is thrown upward with an initial velocity of 16 ft/s from the top of a building 128 ft high, then its height h above the ground t seconds later will be

$$h = 128 + 16t - 16t^2$$

During what time interval will the ball be at least 32 ft above the ground?

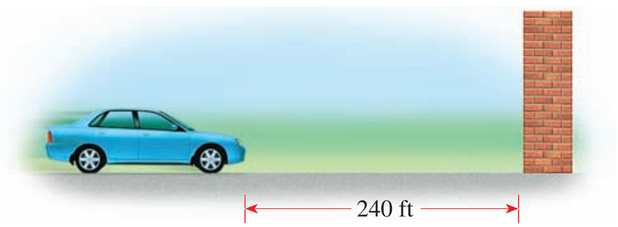


116. Gas Mileage The gas mileage g (measured in mi/gal) for a particular vehicle, driven at v mi/h, is given by the formula $g = 10 + 0.9v - 0.01v^2$, as long as v is between 10 mi/h and 75 mi/h. For what range of speeds is the vehicle's mileage 30 mi/gal or better?

117. Stopping Distance For a certain model of car the distance d required to stop the vehicle if it is traveling at v mi/h is given by the formula

$$d = v + \frac{v^2}{20}$$

where d is measured in feet. Kerry wants her stopping distance not to exceed 240 ft. At what range of speeds can she travel?



118. Manufacturer's Profit If a manufacturer sells x units of a certain product, revenue R and cost C (in dollars) are given by

$$R = 20x$$

$$C = 2000 + 8x + 0.0025x^2$$

Use the fact that

$$\text{profit} = \text{revenue} - \text{cost}$$

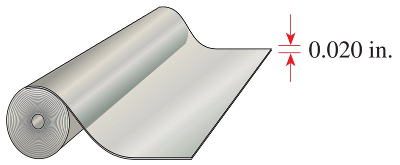
to determine how many units the manufacturer should sell to enjoy a profit of at least \$2400.

119. Fencing a Garden A determined gardener has 120 ft of deer-resistant fence. She wants to enclose a rectangular vegetable garden in her backyard, and she wants the area that is enclosed to be at least 800 ft^2. What range of values is possible for the length of her garden?

120. Thickness of a Laminate A company manufactures industrial laminates (thin nylon-based sheets) of thickness 0.020 in, with a tolerance of 0.003 in.

(a) Find an inequality involving absolute values that describes the range of possible thickness for the laminate.

(b) Solve the inequality you found in part (a).



121. Range of Height The average height of adult males is 68.2 in, and 95% of adult males have height h that satisfies the inequality

$$\left| \frac{h - 68.2}{2.9} \right| \le 2$$

Solve the inequality to find the range of heights.

DISCOVERY ■ DISCUSSION ■ WRITING

122. Do Powers Preserve Order? If $a < b$, is $a^2 < b^2$? (Check both positive and negative values for a and b.) If $a < b$, is $a^3 < b^3$? On the basis of your observations, state a general rule about the relationship between a^n and b^n when $a < b$ and n is a positive integer.

123. What's Wrong Here? It is tempting to try to solve an inequality like an equation. For instance, we might try to solve $1 < 3/x$ by multiplying both sides by x, to get $x < 3$, so the solution would be $(-\infty, 3)$. But that's wrong; for example, $x = -1$ lies in this interval but does not satisfy the original inequality. Explain why this method doesn't work (think about the *sign* of x). Then solve the inequality correctly.

124. Using Distances to Solve Absolute Value Inequalities Recall that $|a - b|$ is the distance between a and b on the number line. For any number x, what do $|x - 1|$ and $|x - 3|$ represent? Use this interpretation to solve the inequality $|x - 1| < |x - 3|$ geometrically. In general, if $a < b$, what is the solution of the inequality $|x - a| < |x - b|$?

SECTION 1.7 ■ PAGE 80

1. (a) $<$ (b) $\le$ (c) $\le$ (d) $>$ **2.** (a) True (b) False
3. (a) $[-3, 3]$ (b) $(-\infty, -3], [3, \infty)$ **4.** (a) < 3 (b) > 3
5. $\{\sqrt{2}, 2, 4\}$ **7.** $\{4\}$ **9.** $\{-2, -1, 2, 4\}$

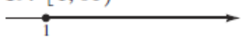
11. $(-\infty, \frac{7}{2}]$

13. $(4, \infty)$

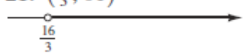
15. $(-\infty, 2]$

17. $(-\infty, -\frac{1}{2})$

19. $[1, \infty)$

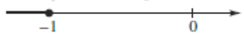


21. $(\frac{16}{3}, \infty)$

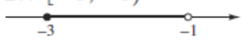


23. $(-\infty, -18)$

25. $(-\infty, -1]$



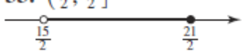
27. $[-3, -1)$



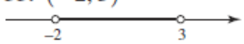
29. $(2, 6)$

31. $[\frac{9}{2}, 5)$

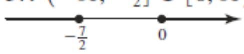
33. $(\frac{15}{2}, \frac{21}{2}]$



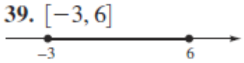
35. $(-2, 3)$



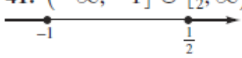
37. $(-\infty, -\frac{7}{2}] \cup [0, \infty)$



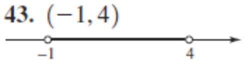
39. $[-3, 6]$



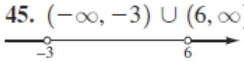
41. $(-\infty, -1] \cup [\frac{1}{2}, \infty)$



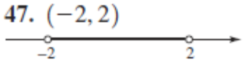
43. $(-1, 4)$



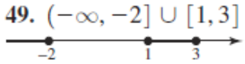
45. $(-\infty, -3) \cup (6, \infty)$



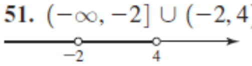
47. $(-2, 2)$



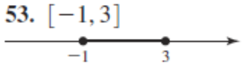
49. $(-\infty, -2] \cup [1, 3]$



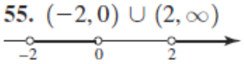
51. $(-\infty, -2] \cup (-2, 4)$



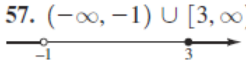
53. $[-1, 3]$



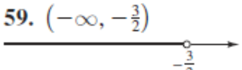
55. $(-2, 0) \cup (2, \infty)$



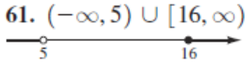
57. $(-\infty, -1) \cup [3, \infty)$



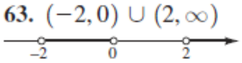
59. $(-\infty, -\frac{3}{2})$



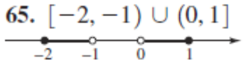
61. $(-\infty, 5) \cup [16, \infty)$



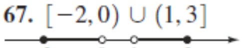
63. $(-2, 0) \cup (2, \infty)$



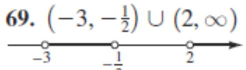
65. $[-2, -1) \cup (0, 1]$



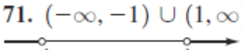
67. $[-2, 0) \cup (1, 3]$



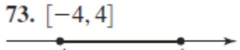
69. $(-3, -\frac{1}{2}) \cup (2, \infty)$



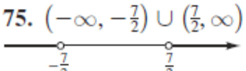
71. $(-\infty, -1) \cup (1, \infty)$



73. $[-4, 4]$



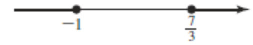
75. $(-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty)$



77. $[2, 8]$

79. $[1.3, 1.7]$

81. $(-\infty, -1] \cup [\frac{7}{3}, \infty)$



83. $(-4, 8)$

85. $(-6.001, -5.999)$

87. $[-\frac{1}{2}, \frac{3}{2}]$

89. $|x| < 3$ **91.** $|x - 7| \ge 5$ **93.** $|x| \le 2$ **95.** $|x| > 3$
97. $|x - 1| \le 3$ **99.** $-\frac{4}{3} \le x \le \frac{4}{3}$ **101.** $x < -2$ or $x > 7$
103. (a) $x \ge \frac{c}{a} + \frac{c}{b}$ (b) $\frac{a - c}{b} \le x < \frac{2a - c}{b}$
105. $68 \le F \le 86$ **107.** More than 200 mi
109. Between 12,000 mi and 14,000 mi
111. (a) $-\frac{1}{3}P + \frac{560}{3}$ (b) From \$215 to \$290
113. Distances between 20,000 km and 100,000 km
115. From 0 s to 3 s **117.** Between 0 and 60 mi/h
119. Between 20 and 40 ft **121.** Between 62.4 and 74.0 in.