When solving equations that involve absolute values, we usually take cases.

EXAMPLE 14 | An Absolute Value Equation

Solve the equation $|2x - 5| = 3$.

SOLUTION By the definition of absolute value, $|2x - 5| = 3$ is equivalent to

$$2x - 5 = 3 \qquad \text{or} \qquad 2x - 5 = -3$$
$$2x = 8 \qquad\qquad 2x = 2$$
$$x = 4 \qquad\qquad x = 1$$

The solutions are $x = 1$, $x = 4$.

NOW TRY EXERCISE **105**

1.5 EXERCISES

CONCEPTS

1. *True or false*?

(a) Adding the same number to each side of an equation always gives an equivalent equation.

(b) Multiplying each side of an equation by the same number always gives an equivalent equation.

(c) Squaring each side of an equation always gives an equivalent equation.

2. Explain how you would use each method to solve the equation $x^2 - 4x - 5 = 0$.

(a) By factoring: ________

(b) By completing the square: ________

(c) By using the Quadratic Formula: ________

3. (a) The solutions of the equation $x^2(x - 4) = 0$ are ________.

(b) To solve the equation $x^3 - 4x^2 = 0$, we ________ the left-hand side.

4. Solve the equation $\sqrt{2x} + x = 0$ by doing the following steps.

(a) Isolate the radical: ________________.

(b) Square both sides: ________________.

(c) The solutions of the resulting quadratic equation are ________.

(d) The solution(s) that satisfy the original equation are ________.

5. The equation $(x + 1)^2 - 5(x + 1) + 6 = 0$ is of ________ type. To solve the equation, we set $W =$ ________. The resulting quadratic equation is ________.

6. The equation $x^6 + 7x^3 - 8 = 0$ is of ________ type. To solve the equation, we set $W =$ ________. The resulting quadratic equation is ________.

SKILLS

7–10 ■ Determine whether the given value is a solution of the equation.

7. $4x + 7 = 9x - 3$
(a) $x = -2$ **(b)** $x = 2$

8. $1 - [2 - (3 - x)] = 4x - (6 + x)$
(a) $x = 2$ **(b)** $x = 4$

9. $\dfrac{1}{x} - \dfrac{1}{x - 4} = 1$
(a) $x = 2$ **(b)** $x = 4$

10. $\dfrac{x^{3/2}}{x - 6} = x - 8$
(a) $x = 4$ **(b)** $x = 8$

11–28 ■ The given equation is either linear or equivalent to a linear equation. Solve the equation.

11. $2x + 7 = 31$

12. $5x - 3 = 4$

13. $\frac{1}{2}x - 8 = 1$

14. $3 + \frac{1}{3}x = 5$

15. $-7w = 15 - 2w$

16. $5t - 13 = 12 - 5t$

17. $\frac{1}{2}y - 2 = \frac{1}{3}y$

18. $\dfrac{z}{5} = \dfrac{3}{10}z + 7$

19. $2(1 - x) = 3(1 + 2x) + 5$

20. $\dfrac{2}{3}y + \dfrac{1}{2}(y - 3) = \dfrac{y + 1}{4}$

21. $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0$

22. $2x - \dfrac{x}{2} + \dfrac{x + 1}{4} = 6x$

23. $\dfrac{1}{x} = \dfrac{4}{3x} + 1$

24. $\dfrac{2x - 1}{x + 2} = \dfrac{4}{5}$

25. $\dfrac{3}{x + 1} - \dfrac{1}{2} = \dfrac{1}{3x + 3}$

26. $\dfrac{4}{x - 1} + \dfrac{2}{x + 1} = \dfrac{35}{x^2 - 1}$

27. $(t - 4)^2 = (t + 4)^2 + 32$

28. $\sqrt{3}x + \sqrt{12} = \dfrac{x + 5}{\sqrt{3}}$

29–42 ■ Solve the equation for the indicated variable.

29. $PV = nRT$; for R

30. $F = G\dfrac{mM}{r^2}$; for m

31. $P = 2l + 2w$; for w

32. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$; for R_1

33. $\dfrac{ax + b}{cx + d} = 2$; for x

34. $a - 2[b - 3(c - x)] = 6$; for x

35. $a^2x + (a - 1) = (a + 1)x$; for x

36. $\dfrac{a + 1}{b} = \dfrac{a - 1}{b} + \dfrac{b + 1}{a}$; for a

37. $V = \frac{1}{3}\pi r^2 h$; for r

38. $F = G\dfrac{mM}{r^2}$; for r

39. $a^2 + b^2 = c^2$; for b

40. $A = P\left(1 + \dfrac{i}{100}\right)^2$; for i

41. $h = \frac{1}{2}gt^2 + v_0 t$; for t

42. $S = \dfrac{n(n + 1)}{2}$; for n

43–54 ■ Solve the equation by factoring.

43. $x^2 + x - 12 = 0$

44. $x^2 + 3x - 4 = 0$

45. $x^2 - 7x + 12 = 0$

46. $x^2 + 8x + 12 = 0$

47. $4x^2 - 4x - 15 = 0$

48. $2y^2 + 7y + 3 = 0$

49. $3x^2 + 5x = 2$

50. $6x(x - 1) = 21 - x$

51. $2x^2 = 8$

52. $3x^2 - 27 = 0$

53. $(3x + 2)^2 = 10$

54. $(2x - 1)^2 = 8$

55–62 ■ Solve the equation by completing the square.

55. $x^2 + 2x - 5 = 0$

56. $x^2 - 4x + 2 = 0$

57. $x^2 - 6x - 11 = 0$

58. $x^2 + 3x - \frac{7}{4} = 0$

59. $2x^2 + 8x + 1 = 0$

60. $3x^2 - 6x - 1 = 0$

61. $4x^2 - x = 0$

62. $x^2 = \frac{3}{4}x - \frac{1}{8}$

63–78 ■ Find all real solutions of the quadratic equation.

63. $x^2 - 2x - 15 = 0$

64. $x^2 + 5x - 6 = 0$

65. $x^2 - 7x + 10 = 0$

66. $x^2 + 30x + 200 = 0$

67. $2x^2 + x - 3 = 0$

68. $3x^2 + 7x + 4 = 0$

69. $3x^2 + 6x - 5 = 0$

70. $x^2 - 6x + 1 = 0$

71. $z^2 - \frac{3}{2}z + \frac{9}{16} = 0$

72. $2y^2 - y - \frac{1}{2} = 0$

73. $4x^2 + 16x - 9 = 0$

74. $0 = x^2 - 4x + 1$

75. $w^2 = 3(w - 1)$

76. $3 + 5z + z^2 = 0$

77. $10y^2 - 16y + 5 = 0$

78. $25x^2 + 70x + 49 = 0$

79–84 ■ Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

79. $x^2 - 6x + 1 = 0$

80. $3x^2 = 6x - 9$

81. $x^2 + 2.20x + 1.21 = 0$

82. $x^2 + 2.21x + 1.21 = 0$

83. $4x^2 + 5x + \frac{13}{8} = 0$

84. $x^2 + rx - s = 0 \quad (s > 0)$

85–108 ■ Find all real solutions of the equation.

85. $\dfrac{1}{x - 1} + \dfrac{1}{x + 2} = \dfrac{5}{4}$

86. $\dfrac{10}{x} - \dfrac{12}{x - 3} + 4 = 0$

87. $\dfrac{x^2}{x + 100} = 50$

88. $\dfrac{1}{x - 1} - \dfrac{2}{x^2} = 0$

89. $\dfrac{x + 5}{x - 2} = \dfrac{5}{x + 2} + \dfrac{28}{x^2 - 4}$

90. $\dfrac{x}{2x + 7} - \dfrac{x + 1}{x + 3} = 1$

91. $\sqrt{2x + 1} + 1 = x$

92. $\sqrt{5 - x} + 1 = x - 2$

93. $2x + \sqrt{x + 1} = 8$

94. $\sqrt{\sqrt{x - 5} + x} = 5$

95. $x^4 - 13x^2 + 40 = 0$

96. $x^4 - 5x^2 + 4 = 0$

97. $2x^4 + 4x^2 + 1 = 0$

98. $x^6 - 2x^3 - 3 = 0$

99. $x^{4/3} - 5x^{2/3} + 6 = 0$

100. $\sqrt{x} - 3\sqrt[4]{x} - 4 = 0$

101. $4(x + 1)^{1/2} - 5(x + 1)^{3/2} + (x + 1)^{5/2} = 0$

102. $x^{1/2} + 3x^{-1/2} = 10x^{-3/2}$

103. $x^{1/2} - 3x^{1/3} = 3x^{1/6} - 9$

104. $x - 5\sqrt{x} + 6 = 0$

105. $|3x + 5| = 1$

106. $|2x| = 3$

107. $|x - 4| = 0.01$

108. $|x - 6| = -1$

APPLICATIONS

109–110 ■ **Falling-Body Problems** Suppose an object is dropped from a height h_0 above the ground. Then its height after t seconds is given by $h = -16t^2 + h_0$, where h is measured in feet. Use this information to solve the problem.

109. If a ball is dropped from 288 ft above the ground, how long does it take to reach ground level?

110. A ball is dropped from the top of a building 96 ft tall.
   (a) How long will it take to fall half the distance to ground level?
   (b) How long will it take to fall to ground level?

111–112 ■ **Falling-Body Problems** Use the formula $h = -16t^2 + v_0 t$ discussed in Example 9.

111. A ball is thrown straight upward at an initial speed of $v_0 = 40$ ft/s.
   (a) When does the ball reach a height of 24 ft?
   (b) When does it reach a height of 48 ft?
   (c) What is the greatest height reached by the ball?
   (d) When does the ball reach the highest point of its path?
   (e) When does the ball hit the ground?

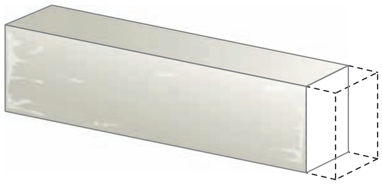
112. How fast would a ball have to be thrown upward to reach a maximum height of 100 ft? [*Hint:* Use the discriminant of the equation $16t^2 - v_0 t + h = 0$.]

113. **Shrinkage in Concrete Beams** As concrete dries, it shrinks—the higher the water content, the greater the shrinkage. If a concrete beam has a water content of w kg/m^3, then it will shrink by a factor

$$S = \frac{0.032w - 2.5}{10{,}000}$$

where S is the fraction of the original beam length that disappears due to shrinkage.
   (a) A beam 12.025 m long is cast in concrete that contains 250 kg/m^3 water. What is the shrinkage factor S? How long will the beam be when it has dried?

(b) A beam is 10.014 m long when wet. We want it to shrink to 10.009 m, so the shrinkage factor should be $S = 0.00050$. What water content will provide this amount of shrinkage?



114. The Lens Equation If F is the focal length of a convex lens and an object is placed at a distance x from the lens, then its image will be at a distance y from the lens, where F, x, and y are related by the *lens equation*

$$\frac{1}{F} = \frac{1}{x} + \frac{1}{y}$$

Suppose that a lens has a focal length of 4.8 cm and that the image of an object is 4 cm closer to the lens than the object itself. How far from the lens is the object?

115. Fish Population The fish population in a certain lake rises and falls according to the formula

$$F = 1000(30 + 17t - t^2)$$

Here F is the number of fish at time t, where t is measured in years since January 1, 2002, when the fish population was first estimated.

(a) On what date will the fish population again be the same as it was on January 1, 2002?

(b) By what date will all the fish in the lake have died?

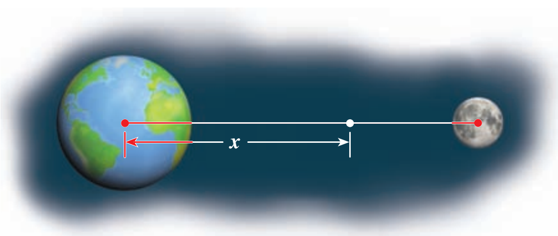
116. Fish Population A large pond is stocked with fish. The fish population P is modeled by the formula $P = 3t + 10\sqrt{t} + 140$, where t is the number of days since the fish were first introduced into the pond. How many days will it take for the fish population to reach 500?

117. Profit A small-appliance manufacturer finds that the profit P (in dollars) generated by producing x microwave ovens per week is given by the formula $P = \frac{1}{10}x(300 - x)$ provided that $0 \le x \le 200$. How many ovens must be manufactured in a given week to generate a profit of \$1250?

118. Gravity If an imaginary line segment is drawn between the centers of the earth and the moon, then the net gravitational force F acting on an object situated on this line segment is

$$F = \frac{-K}{x^2} + \frac{0.012K}{(239 - x)^2}$$

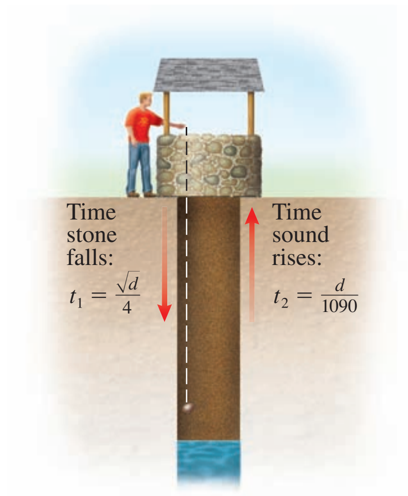
where $K > 0$ is a constant and x is the distance of the object from the center of the earth, measured in thousands of miles. How far from the center of the earth is the "dead spot" where no net gravitational force acts upon the object? (Express your answer to the nearest thousand miles.)



119. Depth of a Well One method for determining the depth of a well is to drop a stone into it and then measure the time it takes until the splash is heard. If d is the depth of the well (in feet) and t_1 the time (in seconds) it takes for the stone to fall, then $d = 16t_1^2$, so $t_1 = \sqrt{d}/4$. Now if t_2 is the time it takes for the sound to travel back up, then $d = 1090t_2$ because the speed of sound is 1090 ft/s. So $t_2 = d/1090$. Thus, the total time elapsed between dropping the stone and hearing the splash is

$$t_1 + t_2 = \frac{\sqrt{d}}{4} + \frac{d}{1090}$$

How deep is the well if this total time is 3 s?



DISCOVERY ■ DISCUSSION ■ WRITING

120. A Family of Equations The equation

$$3x + k - 5 = kx - k + 1$$

is really a **family of equations**, because for each value of k, we get a different equation with the unknown x. The letter k is called a **parameter** for this family. What value should we pick for k to make the given value of x a solution of the resulting equation?

(a) $x = 0$ **(b)** $x = 1$ **(c)** $x = 2$

121. Proof That 0 = 1? The following steps appear to give equivalent equations, which seem to prove that $1 = 0$. Find the error.

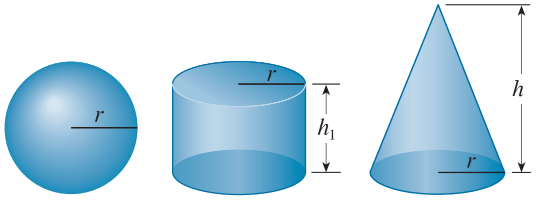
$x = 1$	Given
$x^2 = x$	Multiply by x
$x^2 - x = 0$	Subtract x
$x(x - 1) = 0$	Factor
$\frac{x(x-1)}{x-1} = \frac{0}{x-1}$	Divide by $x - 1$
$x = 0$	Simplify
$1 = 0$	Given $x = 1$

122. Volumes of Solids The sphere, cylinder, and cone shown here all have the same radius r and the same volume V.

(a) Use the volume formulas given on the inside front cover of this book, to show that

$$\tfrac{4}{3}\pi r^3 = \pi r^2 h_1 \quad \text{and} \quad \tfrac{4}{3}\pi r^3 = \tfrac{1}{3}\pi r^2 h_2$$

(b) Solve these equations for h_1 and h_2.



123. Relationship Between Roots and Coefficients The Quadratic Formula gives us the roots of a quadratic equation from its coefficients. We can also obtain the coefficients from the roots. For example, find the roots of the equation $x^2 - 9x + 20 = 0$ and show that the product of the roots is the constant term 20 and the sum of the roots is 9, the negative of the coefficient of x. Show that the same relationship between roots and coefficients holds for the following equations:

$$x^2 - 2x - 8 = 0$$
$$x^2 + 4x + 2 = 0$$

Use the Quadratic Formula to prove that in general, if the equation $x^2 + bx + c = 0$ has roots r_1 and r_2, then $c = r_1 r_2$ and $b = -(r_1 + r_2)$.

124. Solving an Equation in Different Ways We have learned several different ways to solve an equation in this section. Some equations can be tackled by more than one method. For example, the equation $x - \sqrt{x} - 2 = 0$ is of quadratic type. We can solve it by letting $\sqrt{x} = u$ and $x = u^2$, and factoring. Or we could solve for $\sqrt{x}$, square each side, and then solve the resulting quadratic equation. Solve the following equations using both methods indicated, and show that you get the same final answers.

(a) $x - \sqrt{x} - 2 = 0$ quadratic type; solve for the radical, and square

(b) $\dfrac{12}{(x-3)^2} + \dfrac{10}{x-3} + 1 = 0$ quadratic type; multiply by LCD

1.6 Modeling with Equations

Making and Using Models ▶ Problems About Interest ▶ Problems About Area or Length ▶ Problems About Mixtures ▶ Problems About the Time Needed to Do a Job ▶ Problems About Distance, Rate, and Time

Many problems in the sciences, economics, finance, medicine, and numerous other fields can be translated into algebra problems; this is one reason that algebra is so useful. In this section we use equations as mathematical models to solve real-life problems.

▼ Making and Using Models

We will use the following guidelines to help us set up equations that model situations described in words. To show how the guidelines can help you to set up equations, we note them as we work each example in this section.

GUIDELINES FOR MODELING WITH EQUATIONS

1. **Identify the Variable.** Identify the quantity that the problem asks you to find. This quantity can usually be determined by a careful reading of the question that is posed at the end of the problem. Then **introduce notation** for the variable (call it x or some other letter).
2. **Translate from Words to Algebra.** Read each sentence in the problem again, and express all the quantities mentioned in the problem in terms of the variable you defined in Step 1. To organize this information, it is sometimes helpful to **draw a diagram** or **make a table**.
3. **Set Up the Model.** Find the crucial fact in the problem that gives a relationship between the expressions you listed in Step 2. **Set up an equation** (or **model**) that expresses this relationship.
4. **Solve the Equation and Check Your Answer.** Solve the equation, check your answer, and express it as a sentence that answers the question posed in the problem.

SECTION 1.5 ■ PAGE 54

1. **(a)** True **(b)** False (because quantity could be 0) **(c)** False
2. **(a)** Factor into $(x + 1)(x - 5)$, and use the Zero-Product Property. **(b)** Add 5 to each side, then complete the square by adding 4 to both sides. **(c)** Insert coefficients into the Quadratic Formula. **3.** **(a)** 0, 4 **(b)** factor **4.** **(a)** $\sqrt{2x} = -x$ **(b)** $2x = x^2$ **(c)** 0, 2 **(d)** 0
5. quadratic; $x + 1$; $W^2 - 5W + 6 = 0$
6. quadratic; x^3; $W^2 + 7W - 8 = 0$ **7.** **(a)** No **(b)** Yes
9. **(a)** Yes **(b)** No **11.** 12 **13.** 18 **15.** -3 **17.** 12
19. $-\frac{3}{4}$ **21.** 30 **23.** $-\frac{1}{3}$ **25.** $\frac{13}{3}$ **27.** -2 **29.** $R = \dfrac{PV}{nT}$
31. $w = \dfrac{P - 2l}{2}$ **33.** $x = \dfrac{2d - b}{a - 2c}$ **35.** $x = \dfrac{1 - a}{a^2 - a - 1}$
37. $r = \pm\sqrt{\dfrac{3V}{\pi h}}$ **39.** $b = \pm\sqrt{c^2 - a^2}$
41. $t = \dfrac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$ **43.** $-4, 3$ **45.** 3, 4 **47.** $-\frac{3}{2}, \frac{5}{2}$
49. $-2, \frac{1}{3}$ **51.** ± 2 **53.** $-\dfrac{2 \pm \sqrt{10}}{3}$ **55.** $-1 \pm \sqrt{6}$
57. $3 \pm 2\sqrt{5}$ **59.** $-2 \pm \frac{\sqrt{14}}{2}$ **61.** $0, \frac{1}{4}$ **63.** $-3, 5$ **65.** 2, 5
67. $-\frac{3}{2}, 1$ **69.** $-1 \pm \frac{2\sqrt{6}}{3}$ **71.** $\frac{3}{4}$ **73.** $-\frac{9}{2}, \frac{1}{2}$
75. No real solution **77.** $\dfrac{-8 \pm \sqrt{14}}{10}$ **79.** 2 **81.** 1
83. No real solution **85.** $-\frac{7}{5}, 2$ **87.** $-50, 100$ **89.** -4
91. 4 **93.** 3 **95.** $\pm 2\sqrt{2}, \pm\sqrt{5}$ **97.** No real solution
99. $\pm 3\sqrt{3}, \pm 2\sqrt{2}$ **101.** $-1, 0, 3$ **103.** 27, 729
105. $-2, -\frac{4}{3}$ **107.** 3.99, 4.01 **109.** 4.24 s
111. **(a)** After 1 s and $1\frac{1}{2}$ s **(b)** Never **(c)** 25 ft
(d) After $1\frac{1}{4}$ s **(e)** After $2\frac{1}{2}$ s **113.** **(a)** 0.00055, 12.018 m
(b) 234.375 kg/m^3 **115.** **(a)** After 17 yr, on Jan. 1, 2019
(b) After 18.612 yr, on Aug. 12, 2020 **117.** 50 **119.** 132.6 ft